Searching Forever After*

Yair Antler[†] and Benjamin Bachi[‡]

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Abstract

Modern matching markets are becoming less frictional yet more complex. We study the implications of these phenomena through the lens of a two-sided search model in which agents' reasoning is coarse. In equilibrium, the most desirable agents behave as if they were fully rational, while, for other agents, coarse reasoning results in overoptimism with regard to their prospects in the market. Consequently, they search longer than optimal. Moreover, agents with intermediate match values may search indefinitely while all other agents eventually marry. We show that the share of eternal singles converges monotonically to 1 as search frictions vanish.

1 Introduction

Modern search technologies present new opportunities for individuals who are looking for a partner. For instance, mobile applications such as Tinder and Bumble, and online dating sites such as OkCupid and Plenty of Fish, allow individuals to find a partner in the swipe of a finger. These new technologies have reduced search costs and thickened matching markets, which enables individuals to meet a large number of potential matches in a short span of time.

Choosing a partner is one of the most important decisions in a person's life. This decision typically entails comparing a specific potential partner to a risky outside option, that is, continuing to search without knowing for how long or with whom one will

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[†]Coller School of Management, Tel Aviv University. *E-mail:* yair.an@gmail.com.

[‡]Department of Economics, University of Haifa. *E-mail:* bbachi@econ.haifa.ac.il

partner eventually. Assessing this outside option requires understanding other people's behavior, which can be challenging, especially in light of the much wider array of options the new search technologies bring. Some individuals can be overwhelmed by these new possibilities. For example, according to a survey by Pew Research Center (2016), "One-third of people who have used online dating have never actually gone on a date with someone they met on these sites."

This paper studies how advances in search technology affect individual decision making, length of search, and induced matching in the marriage market. To this end, we study a model of two-sided search with vertical differentiation and non-transferable utility. This framework has proved useful in understanding decentralized matching markets such as the marriage and labor markets (see Chade, Eeckhout, and Smith, 2017, for a comprehensive review). In this framework, agents are matched at random each period and decide whether to accept the match or continue to search. It is typically assumed that the participants are fully rational and can perfectly assess the prospect of remaining single and continuing to search. However, in light of the overwhelmingly large array of options that are ubiquitous in modern matching markets and the complexity of the induced search problem, it makes sense to think that individuals use heuristics and simplified models to assess this prospect.

We depart from the rational expectations paradigm by relaxing the assumption that agents have a perfect understanding of the mapping from the other agents' characteristics to their behavior. The literature offers two main notions that capture the idea that agents have an imperfect perception of this mapping: the partially cursed equilibrium (Eyster and Rabin, 2005) and the analogy-based expectation equilibrium (Jehiel, 2005). Both notions allow for various levels of coarse strategic reasoning. In our setting, these notions capture different biases and induce different beliefs. Our framework enables us to adapt each of these behavioral notions while keeping the analysis tractable. We find that, despite their differences, both notions lead to similar results.

We characterize the equilibria of the model and show that, except for the most desirable agents, who behave as if they were fully rational, all other agents are overoptimistic with regard to the prospect of remaining single and continuing to search. The agents' overoptimism follows from two reasons: overestimating the payoff they will obtain in a future marriage and underestimating the time it will take them to get married.

This overoptimism has significant implications as, except for the least desirable agents who accept all potential partners, agents who overvalue the prospect of remaining single reject agents whom a rational agent would accept. In a two-sided market, overoptimistic agents not only search longer than rational agents, but also impose a negative externality on agents on the other side of the market and cause them to search longer as they make it more difficult for them to find a partner. In equilibrium, this effect leads to a delay in matching. In fact, as long as the discount factor is not too small, in every symmetric equilibrium, there are agents with intermediate match values (i.e., moderately desirable agents) who search indefinitely and remain single forever. By contrast, agents with lower or higher match values marry in finite time. Thus, the extent to which overoptimism harms the agents is nonmonotonic in their match values.

We show that for any level of coarseness (or, in Eyster and Rabin's terminology, any degree of partial cursedness), when search frictions become less intense, the share of agents who search indefinitely weakly increases. Moreover, it converges to 1 when search frictions vanish. This result implies that even a slight departure from the rational expectations model can lead to a market failure when search frictions vanish. Thus, technological improvements that result in faster and more efficient search, which can enhance individuals' welfare if they are fully rational, can degrade it if they are not.

The intuition for the market failure is as follows. If agents falsely believe that "top" agents are achievable, they prefer to wait for these agents. When the technology allows potential partners to meet more frequently, their willingness to wait for a top agent increases and they become more selective. Eventually, when the search technology improves substantially, agents become too selective and reject agents of their own caliber or lower. For similar reasons, they are rejected by agents of their own caliber or higher. As a result, they search indefinitely and never marry.

Our results are quite different from the results under the rational expectations model, in which, when frictions vanish, the equilibrium converges to a stable matching in the sense of Gale and Shapley (1962). This contrast highlights that the common wisdom that matchings will be stable when participants "have a very good idea of one another's preferences and have easy access to each other" (Roth and Sotomayor, 1990) hinges on the participants also having a good idea of each other's behavior and expectations. While counter-intuitive, the finding that fewer matches are formed when the market becomes less frictional is consistent with recent empirical findings: Fong (2019) shows that when more men and women join a dating platform, people in the market become more selective and the number of matches per individual goes down.

Related Literature

Our paper contributes to a large body of literature on matching with frictions (see MacNamara and Collins, 1990; Burdett and Coles, 1997; Eeckhout, 1999; Bloch and Ryder, 2000; Shimer and Smith, 2000; Chade, 2001; Adachi, 2003; Chade, 2006; Smith, 2006). This literature focuses on the properties of induced matching under various assumptions on search frictions, match payoffs, search costs, and the ability to transfer utility. It shows that when utility is nontransferable and frictions vanish, induced matching converges to efficient matching.[1]

We assume that the agents' reasoning is coarse and borrow the behavioral models of Eyster and Rabin (2005) and Jehiel (2005). Similar ideas have been applied in various contexts. For example, Piccione and Rubinstein (2003) study intertemporal pricing, where consumers think in terms of a coarse representation of the equilibrium price distribution.[2] In the context of consumer search, Gamp and Krähmer (2019a) analyze a model in which a share of consumers do not distinguish between deceptive and candid products nor can they infer quality from price. In Gamp and Krähmer (2019b), a share of the consumers misestimate the correlation between price and quality and, as a result, search excessively in order to find a high-quality product at a low price, falsely believing that this combination exists. These false expectations stimulate competition between fully rational sellers and the effect is most intense when the consumers' search costs (or level of misestimation) are intermediate.[3]

We find that, in the matching with frictions framework, coarse reasoning leads to selection neglect in equilibrium. Esponda (2008) proposes an equilibrium model of selection neglect and shows that traders who do not account for selection can exacerbate adverse selection problems. In Jehiel (2018), entrepreneurs decide whether or not to invest in a project based on feedback from implemented projects. The entrepreneurs ignore the lack of feedback from non-implemented projects, which, on average, are

[1] Lauermann and Nöldeke (2014) find conditions under which this result holds without vertical heterogeneity.

[2] Other applications are Jehiel (2010) in the context of auctions, Eyster and Piccione (2013), Steiner and Stewart (2015), Kondor and Kőszegi (2017), and Eyster et al. (2019) in the context of trade in financial markets, and Antler (2019) in the context of pyramid schemes.

[3] In these models, agents can be viewed as if they were using a simplified representation of the world to form their expectations. For a comprehensive review of equilibrium models in which individuals interpret data by means of a misspecified causal model see Spiegler (2019).

inferior to implemented ones. As a result they become overoptimistic and implement projects in cases where it is suboptimal to do so.

To the best of our knowledge there are a limited number of theoretical papers that relax the full rationality assumption in the context of matching. Eliaz and Spiegler (2013) analyze a search and matching model where agents exhibit "morale hazard." The behavioral assumption in that paper pertains to the agents' preferences rather than to their beliefs. In the context of centralized matching, a recent strand of the literature assumes that agents' preferences are non-standard in the context of centralized matching (see, e.g, Antler, 2015; Fernández, 2018; Dreyfuss et al., 2019; Meisner and von Wangenheim, 2019).

Our departure from the rational expectations setting is in line with empirical evidence that people neglect correlations when problems become more complex (Enke and Zimmerman, 2017). In the context of centralized school matching, Shorer et al. (2019) find that students tend to neglect correlation between schools' tastes and priorities. In the context of courtship, Fisman et al. (2006) find that men exhibit behavior consistent with choice overload, and Francesconi and Lenton (2010) document similar findings on both sides of the market.

The paper proceeds as follows. Section 2 present the baseline model and benchmark results. Section 3 introduces the behavioral models and their analysis. Section 4 concludes. All proofs are relegated to the Appendix.

2 The Baseline Model

There is a set of men $\mathbf{M}$ and a set of women $\mathbf{W}$, each containing a unit mass of agents. Each agent is characterized by a number, which, following Burdett and Coles (1997), we refer to as the agent's pizzazz. The agents' pizzazz is distributed on the interval $[\underline{v}, \overline{v}]$, $\underline{v} > 0$, according to an atomless continuous distribution F. We denote the corresponding density by f and often refer to an agent with pizzazz v as agent v.

In each period, a measure $\mu > 0$ of men and a measure μ of women are drawn uniformly at random. These men and women are then randomly matched with each other. When a pair of agents are matched, they immediately observe each other's pizzazz and choose whether to accept or reject the match. If both agents accept, then they marry, exit the market, and are replaced by two agents with identical characteristics who start searching in the next period. Otherwise, the match is dissolved, and the agents return to the market and continue their search in the next period. When agent v marries agent w, the latter obtains a payoff of v and the former obtains a payoff of w. All agents discount the future at a rate $\delta < 1$ and obtain no payoff when single. We refer to δ and μ as search frictions and assume that the agents maximize their expected discounted payoff given their beliefs.

A (stationary) strategy for agent v, $\sigma_v(\cdot) : [\underline{v}, \overline{v}] \to \{1, 0\}$, is a mapping from pizzazz of agents on the other side of the market to a decision whether to accept or reject a match.[4] We say that agent v uses a cutoff strategy if there exists $\hat{a}_v$ such that agent v accepts matches with agents whose pizzazz is at least $\hat{a}_v$ and rejects all others. Throughout the analysis, we assume that an agent who is indifferent whether to accept a match or not accepts it, which implies that the agents use cutoff strategies. For each agent v and profile σ, let $A_v(\sigma) = \{w|\sigma_w(v) = 1\}$ be the set of agents who accept a match with v and let $a_v(\sigma) = max\,\{\underline{v}, \sup(A_v(\sigma))\}$. We often refer to $A_v(\sigma)$ and $a_v(\sigma)$ as agent v's opportunity set and opportunity value, respectively. When there is no risk of confusion, we omit the dependence on σ from A_v and a_v.

Throughout the analysis, we focus on symmetric equilibria, namely, equilibria in which women and men with the same pizzazz use the same strategy. This symmetry assumption greatly simplifies the notation and makes the exposition clearer. However, the key results and intuitions remain valid when this assumption is dropped (or when the distributions of men's and women's pizzazz are different). We discuss and explain the minor differences at the end of Section 3.1.3, after presenting our results.

Benchmark Results: Full Rationality

We now provide a "rational expectations" benchmark. As the analysis follows from well-known results in the matching with frictions literature, we omit the formal proofs. Proposition 1 is a classic block segregation result (see, e.g., MacNamara and Collins, 1990; Coles and Burdett, 1997; Eeckhout, 1999; Bloch and Ryder, 2000; Chade, 2001; Smith, 2006).

Proposition 1 *There exist numbers $\overline{v} = v^0 > v^1 > v^2 > ... > v^N = \underline{v}$ such that, in the unique equilibrium, every agent $v \in [v^{j+1}, v^j)$ uses the acceptance cutoff v^{j+1}.*

In equilibrium, the agents are partitioned into *classes*, such that agents who belong to the same class use the same acceptance cutoff and have the same opportunity value.

[4] Abusing notation, we can think of σ_v as a subset of agents on the other side of the market whom v accepts. Then, $\sigma : [\underline{v}, \overline{v}] \to \mathcal{P}([\underline{v}, \overline{v}])$ is a correspondence, which we assume to be measurable.

All agents are accepted by members of their class and rejected by all members of higher classes. Thus, agents marry within their class and no agent remains single forever.

By Proposition 1, in equilibrium, every agent's pizzazz is strictly greater than her/his acceptance cutoff, except for agents at the lower bound of a class. When search frictions vanish, the induced matching converges to the unique stable matching (see Eeckhout, 1999; Bloch and Ryder, 2000; Adachi, 2003), which implies that married couples have the same pizzazz. Thus, when search frictions vanish, the classes shrink and almost all of the agents' acceptance cutoffs increase. These increases can be interpreted as an increase in the agents' welfare.[5]

3 Coarse Reasoning in the Matching Market

Consider an agent who faces a decision whether to accept or reject a match. Since this decision has implications only when there is mutual consent, the agent essentially compares the known payoff from marrying the partner to the risky option of remaining single and continuing to search. Assessing the latter option requires the agent to predict the future behavior of agents of the opposite sex.

A fully rational agent would form an accurate prediction as such agents understand the other agents' behavior perfectly: they know who finds them acceptable and who does not. Our agent, however, has a coarse perception of the other agents' behavior. (S)he understands the rate at which (s)he is accepted by potential partners, but does not discern exactly who finds her/him acceptable and who does not. Essentially, our agent under-appreciates the correlation between the other agents' pizzazz and their behavior.

The two most prominent approaches that capture this idea are the partially cursed equilibrium (Eyster and Rabin, 2005) and the analogy-based expectation equilibrium (Jehiel, 2005). In Section 3.1, we take the first approach, which captures the idea that agents make small mistakes but with respect to the whole population. In Section 3.2, we take the second approach, which captures the idea that agents make mistakes but with respect to only a small fraction of the population. We show that, under both approaches, when search frictions vanish, a small behavioral friction leads to radically different results than the perfectly assortative matching under the rational expectations

[5] In equilibrium, every agent v's acceptance cutoff, $\hat{a}_v$, is equal to v's continuation payoff. When time is continuous, $\hat{a}_v$ is also v's expected payoff when v enters the game. It is also the case in discrete time, unless v gets the first sample for free.

model.

3.1 The Partially Cursed Equilibrium

The notion of partial cursedness was developed by Eyster and Rabin (2005) for Bayesian games. We now adapt this notion to our setting. Before doing so, it is perhaps useful to first understand the notion of "full cursedness" in the context of two-sided matching. Fully cursed agents believe that every agent of the opposite sex accepts them as a partner with a probability that equals the share of agents on the other side of the market who accept them. For example, a fully cursed man who is accepted only by women whose pizzazz is lower than the median believes that each and every woman accepts him with probability 0.5 regardless of her pizzazz.

Partially cursed agents understand that the other agents' behavior depends on their pizzazz but, unlike fully cursed agents, they do not understand to what extent. Specifically, a partially cursed agent v believes that an agent w on the other side of the market will accept her/him as a partner with a probability that is a convex combination of the true probability with which w accepts v and the average rate at which v is accepted by the general population.

Given a strategy profile σ, the true probability that agent w accepts agent v is $\sigma_w(v)$. The average rate at which the general population accepts agent v is

$$\int_{\underline{v}}^{\overline{v}} \sigma_x(v) f(x) dx. \tag{1}$$

Thus, a partially cursed agent v believes that agent w will accept her/him as a partner with probability

$$\gamma_v(w) = \psi \int_{\underline{v}}^{\overline{v}} \sigma_x(v) f(x) dx + (1-\psi)\sigma_w(v), \tag{2}$$

where ψ represents the magnitude of the agents' mistakes. When $\psi = 0$, agents have rational expectations. At the other extreme, when $\psi = 1$, agents are fully cursed and neglect the correlation between the other agents' pizzazz and their behavior. When $\psi \in (0, 1)$, agents understands that different agents' decisions may depend on their pizzazz but under-estimates this relation. Thus, ψ can be thought of as a *behavioral friction.*

As an illustration, suppose that the median pizzazz in the population is w_m and that $A_v = \{w | w < w_m\}$ for some man v. If v were fully rational, then he would expect a woman w to accept a match with him if and only if $w < w_m$. A partially cursed man v with $\psi = 0.1$ expects women whose pizzazz is greater than w_m to accept him with probability 0.05, and women whose pizzazz is lower than w_m to accept him with probability 0.95. Thus, v overestimates the probability of being accepted by women whose pizzazz is high and underestimates the probability of being accepted by women whose pizzazz is low.

Agent v's (perceived) expected payoff at the beginning of each period conditional on using an acceptance cutoff $\hat{a}_v$ and beliefs $\gamma(\cdot)$ is

$$U_v = \mu \int_{\hat{a}_v}^{\overline{v}} \gamma_v(x) x f(x) dx + \delta \Big(1 - \mu \int_{\hat{a}_v}^{\overline{v}} \gamma_v(x) f(x) dx \Big) U_v. \tag{3}$$

Rearranging yields

$$U_v = \frac{\mu \int_{\hat{a}_v}^{\overline{v}} \gamma_v(x) x f(x) dx}{1 - \delta(1 - \mu \int_{\hat{a}_v}^{\overline{v}} \gamma_v(x) f(x) dx)} \tag{4}$$

Definition 1 *A strategy profile σ forms a partially cursed equilibrium if, for each $v \in M \cup W$, σ_v is optimal given γ_v.*

We denote the (perceived) expected payoff conditional on using an optimal acceptance cutoff by $U_v^\star$ and refer to $\delta U_v^\star$ as agent v's *continuation value*.

Agent v accepts a match with agent w if and only if $w \geq \delta U_v^\star$, implying that, in equilibrium, $\hat{a}_v = max\,\{\delta U_v^\star, \underline{v}\}$. The following lemma uses this property to establish that, in equilibrium, agents with higher pizzazz have higher standards.

Lemma 1 *In equilibrium, $\hat{a}_v$ and a_v are weakly increasing in v.*

Lemma 1 implies that if $w \in A_v$, then $w' \in A_v$ for every $w' < w$. Thus, agent v is rejected by every agent whose pizzazz is greater than a_v and accepted by every agent whose pizzazz is lower than a_v. We now use the monotonicity result to understand who marries in equilibrium.

3.1.1 Who Marries in Equilibrium?

Under the conventional rational expectations model, if the distribution of agents on both sides of the market is symmetric, then all agents marry in finite time. In our model, the agents' coarse reasoning makes some of them too selective to marry. The next definition will be useful in understanding why this happens and who these *eternal singles* are.

Definition 2 *Let s_v be the perceived discounted expected payoff of an agent whose opportunity value is v and who uses the cutoff v.*

Under partial cursedness, if $a_w = v$, then $\gamma_w(x) = \psi F(v)$ for any $x > v$. Thus,

$$s_v = \frac{\delta\mu \int_v^{\overline{v}} \psi F(v) x f(x) dx}{1 - \delta(1 - \mu \int_v^{\overline{v}} \psi F(v) f(x) dx)} = \frac{\delta\mu\psi F(v)(1 - F(v))E[x|x > v]}{1 - \delta(1 - \mu\psi F(v)(1 - F(v)))} \tag{5}$$

Note that s_v depends only on the primitives of the model. The next lemma uses s_v to provide a necessary and sufficient condition for marriage in a symmetric equilibrium.

Lemma 2 *Agent v marries in a symmetric equilibrium if and only if*

$$v > s_v. \tag{6}$$

Lemma 2 establishes a necessary and sufficient condition for an agent to marry in symmetric equilibria. When Condition 6 is not satisfied, it means that given an opportunity value v, an agent will prefer remaining single to marrying agent v. An agent v for whom $s_v \geq v$ exhibits a "Groucho Marx" type of behavior, as v is unwilling to marry agents who are willing to marry v.

In order to gain intuition for the condition's necessity, observe that s_v is weakly lower than the continuation value of a woman whose opportunity value is v. By the monotonicity of the acceptance cutoffs, whenever man v is willing to marry a woman, her opportunity value is at least v. Hence, when $s_v > v$, every woman that man v finds acceptable prefers remaining single to marrying him. Therefore, v cannot marry in equilibrium.[6]

To see why Condition 6 is also sufficient, note that in a symmetric equilibrium, it must be that $\hat{a}_v \geq v \geq a_v$ or $\hat{a}_v \leq v \leq a_v$. If $a_v > v$, agent v marries in a symmetric equilibrium as there are agents who accept v and whom v finds acceptable as well. Condition 6 essentially implies that if $a_v \leq v$, then v is willing to accept agents whose pizzazz is lower than v, i.e., $\hat{a}_v < v$. Since $\hat{a}_v < v$ and $a_v < v$ cannot both hold in a

[6] This argument does not depend on whether the equilibrium is symmetric or not.

symmetric equilibrium, it follows that $a_v = v > \hat{a}_v$ in this case. Hence, when Condition 6 holds, there are agents who accept v and whom v finds acceptable as well in every symmetric equilibrium.

Condition 6 allows us to understand who will marry in a partially cursed equilibrium. As (5) is continuous and $s_{\overline{v}} = s_{\underline{v}} = 0$, agents with *extreme pizzazz values always marry* in finite time while agents with intermediate pizzazz may search indefinitely. We now use Condition 6 to study the effect of the matching frictions, μ and δ, and the behavioral friction ψ on the share of eternal singles.

Proposition 2 *The share of agents who marry in a symmetric equilibrium weakly decreases in δ, μ, and ψ. Moreover, it converges to 0 as δ converges to 1.*

Proposition 2 establishes that the share of eternal singles increases when the market becomes less frictional. Note that an agent v marries in equilibrium only if (s)he is accepted by agents with pizzazz lower than v. Thus, a share of at least $F(v)$ agents accept v in such a case. Hence, agent v believes that (s)he will be accepted by every agent with a probability of at least $\psi F(v) > 0$. As search frictions vanish, v expects to encounter more and more agents with high pizzazz and, since v thinks that all agents are achievable with probability at least $\psi F(v)$, agent v will never accept an agent of her/his own caliber or lower. For similar reasons, v will never be accepted by agents of her/his caliber or higher, which makes it impossible for v to marry.

3.1.2 Overoptimism and Oversearch

In the previous section, we showed that some agents in our model may search indefinitely and never leave the market. This suggests that even when they do marry, agents search longer than is optimal given the other agents' behavior. The next result shows that this is indeed the case.

Proposition 3 *There exist pizzazz values $v^1 < \overline{v}$ and $v^2 > \underline{v}$ such that if $v \geq v^1$ or $v < v^2$, then agent v behaves as if (s)he were fully rational. If $v^2 \leq v < v^1$, then agent v searches longer than a rational agent would. Moreover, $[v^1, \overline{v}]$ is the top class in Proposition 1.*

Agents who are accepted by all other agents are unaffected by cursedness as all other agents treat them equally. They correctly estimate both the expected time it will take them to marry and their future spouse's expected pizzazz. Thus, they behave as if they were fully rational.

All other agents are overoptimistic with regard to their prospects in the market. They overestimate the expected pizzazz of their future spouse, underestimate the time it will take them to get married, and, as a result, use an acceptance cutoff that is higher than optimal.[7] In other words, overoptimistic agents reject some matches that a fully rational agent would accept.

Our agents underestimate the time it will take them to marry because they overestimate the rate of mutual acceptance: they accept agents on the other side of the market who have higher standards than the general population, but do not fully account for this selection. Thus, unless an agent is accepted by all agents or accepts all other agents (in which case, there is no selection), the agent overestimates the rate of mutual acceptance.

Our agents also overestimate the expected desirability of their future spouse. They assign a positive probability to being accepted by highly desirable agents who actually reject them and a too low probability to being accepted by low-pizzazz agents who do accept them. Thus, when assessing the expected desirability of their eventual partners, they put too much weight on partners with relatively high pizzazz and too little weight on partners with relatively low pizzazz.

3.1.3 Characterization and Existence

In this subsection, we focus on the structure and existence of the equilibria of the model. We construct a symmetric equilibrium in which there is block segregation. Our construction shows that, in general, there are an infinite number of such equilibria. However, by Condition 6, the set of agents who marry in equilibrium is unique.

In constructing the symmetric equilibria, we use the fact that Condition 6 allows us to partition $[\underline{v}, \overline{v}]$ into maximal intervals in which either all agents marry or none do. We refer to these intervals as marriage intervals and singles intervals, respectively. The transition between intervals occurs at points v such that $s_v = v$. We treat each interval separately and partition it into a potentially infinite number of *classes*, in which all agents share the same acceptance cutoff and opportunity value.

As in the rational case, a top class exists and it is possible to construct a sequence

[7]There is one exception: agents who accept matches with every other agent (i.e., agents whose pizzazz is lower than v^2) never reject matches a fully rational agent would accept.

of classes starting from this class. However, unlike in the rational case, the sequence will not necessarily cover $[\underline{v}, \overline{v}]$. When the sequence does not cover $[\underline{v}, \overline{v}]$, it converges to the highest pizzazz v such that $s_v = v$. That is, the classes cover only the top marriage interval.

The main challenge in the proof is that, unlike in the top marriage interval, in any other interval there is no upper class from which we can start the construction. Nevertheless, we show that it is possible to define an arbitrary initial class in the interior of each interval and construct two unique sequences of classes on each of the initial class's sides. The sequences cover the interval and converge to its end points. The freedom in defining the initial class implies that there are an infinite number of equilibria whenever $[\underline{v}, \overline{v}]$ is partitioned into more than one interval.

Formally, by Condition 6, we can partition $[\underline{v}, \overline{v}]$ into maximal intervals in which agents either eventually marry or remain single forever. We say that L is a *marriage interval* if L is a maximal interval such that $s_l < l$ for all $l \in L$. An interval L is said to be a *singles interval* if either L is a maximal interval such that $s_l > l$ for all $l \in L$, or $s_l = l$ for all $l \in L$. In the latter case, L is often a singleton. Denote $[\underline{l}, \overline{l}] := cl(L)$.

We say that C is a *class* if for every $v, w \in C$, it holds that $a_v = a_w \in \{\underline{c}, \overline{c}\}$ and $\hat{a}_v = \hat{a}_w \in \{\underline{c}, \overline{c}\}$, where $[\underline{c}, \overline{c}] = cl(C)$ and $\underline{c} \neq \overline{c}$. We refer to classes contained in marriage intervals as *marriage classes* and to classes contained in singles intervals as *singles classes.*

The following lemma is key in establishing that, in equilibrium, if an interval contains one class, then it is covered by classes.

Lemma 3 *In equilibrium, if an interval L contains a class C, then (i) unless $\overline{c} = \overline{v}$, L contains a unique class C' such that $\overline{c} = \underline{c}'$, and (ii) unless $\underline{c} = \underline{v}$, L contains a unique class C'' such that $\overline{c''} = \underline{c}$.*

By Lemma 3, if an interval L contains a finite number of classes, then $L = [\underline{v}, \overline{v}]$. Moreover, an infinite sequence of classes must converge to some v satisfying $s_v = v$. That is, for any maximal interval L, it holds that $\underline{l} \in \{\underline{v}, s_{\underline{l}}\}$ and $\overline{l} \in \{\overline{v}, s_{\overline{l}}\}$. The next corollary follows immediately.

Corollary 1 *If an interval contains one class, then it is covered by classes.*

Due to the search frictions, the agents' continuation values are bounded by $\delta\overline{v}$. Thus, in equilibrium, there are two sets of agents, one on each side of the market, who are accepted by every agent of the opposite sex. All of these agents have the same continuation value and, therefore, they use the same acceptance cutoff, which, in turn, defines the set of agents who are accepted by all the other agents on the other side of the market. These agents form the top class and are uniquely determined by the primitives of the model. Thus, Corollary 1 and Proposition 3 imply the following corollary.

Corollary 2 *In equilibrium, the top marriage interval is covered by classes in a unique manner.*

If δ is sufficiently low such that $s_v < v$ for all v, implying that all agents marry in equilibrium, then, by Corollary 2, the equilibrium is unique. Otherwise, the equilibrium is uniquely defined in the top marriage interval and only in that interval.

In the following proposition, we show equilibrium existence by construction.

Proposition 4 *There exists $\overline{\delta}$ such that if $\delta < \overline{\delta}$, then there exists a unique equilibrium and all agents marry. If $\delta > \overline{\delta}$, then there exist an infinite number of equilibria, in each of which there is a set of agents who remain single, and this set of eternal singles is the same in all of these equilibria.*

In the symmetric equilibria we constructed, the agents are partitioned into classes. However, in addition to these equilibria, when $\delta \geq \overline{\delta}$, there are equilibria in which the top marriage interval is covered by classes while in every other interval there are no classes at all. In general, in every interval but the top one, it is possible to construct equilibrium strategies where the cutoffs and opportunity values are continuous in the agents' types. Thus, it is possible to construct equilibria in which close "types" never have completely disjointed sets of potential partners except in the very top marriage interval.

Comment: Symmetry in the Model

Throughout the analysis, we imposed symmetry along three dimensions: we focused on symmetric equilibria, assumed that the men's and women's pizzazz values are drawn from identical distributions, and studied the case where the men's and women's level of strategic sophistication is identical. These assumptions allowed us to convey the main messages succinctly. However, our key insights are not sensitive to these assumptions. Since the implications of relaxing the first two types of symmetry are similar, we assume that agents' pizzazz are drawn from identical distributions and focus our discussion on asymmetric equilibria and different levels of strategic sophistication.

Asymmetric Equilibria

The cornerstone of our analysis of symmetric equilibria was the necessary and sufficient condition for marriage, $s_v > v$. In asymmetric equilibria, this condition is necessary for marriage, but it is no longer sufficient. Thus, the agents who marry in a symmetric equilibrium form a superset of the agents who marry in an asymmetric equilibrium.

When $s_v > v$, agent v cannot marry in asymmetric equilibria. To see why, note that if $s_v > v$ and man (woman) v accepts a match with woman (man) w, then all men (women) with pizzazz lower than v accept w and so $a_w \geq v$. When $s_v > v$ and $a_w \geq v$, agent w prefers continuing searching to marrying v. Thus, agent v is rejected by every agent whom v accepts and remains single forever. Moreover, when the market becomes less frictional, more agents remain single (since s_v increases in δ, μ, and ψ).

In order to illustrate that $s_v < v$ is not sufficient for marriage in asymmetric equilibria, let δ be such that $s_v = v$ for some v and denote that lowest such v by $v^\star$. Note that $v^\star > \underline{v}$ since $s_{\underline{v}} = 0$. Set $\hat{a}_v = v^\star$ for all women with $v \in [\underline{v}, v^\star]$ and $\hat{a}_v = \underline{v}$ for all men with $v \in [\underline{v}, v^\star]$. For all other agents, define acceptance cutoffs as in one of the symmetric equilibria constructed in the proof of Proposition 4. Note that (i) $a_v = v^\star$ for all women with $v \in [\underline{v}, v^\star]$, which implies that $\hat{a}_v = v^\star$ is optimal for these women and (ii) $a_v = \underline{v}$ for all men with $v \in [\underline{v}, v^\star]$, which implies that $\hat{a}_v = \underline{v}$ is optimal for these men. As in Proposition 4, agents whose pizzazz is lower than $v^\star$ accept all agents whose pizzazz is greater than $v^\star$, which makes the higher-pizzazz agents' behavior optimal in the present case as well. Hence, the strategy profile we constructed is an equilibrium in which low-pizzazz agents never marry, unlike in a symmetric equilibrium, in which these agents always do.

This example highlights a more general property: in a symmetric equilibrium, agents are partitioned into marriage and singles intervals, based on whether $s_v \geq v$ or $s_v < v$ in each interval. In an asymmetric equilibrium, it is possible to turn every marriage interval into a singles interval (except for the top one, in which the agents' behavior is pinned down by its top class).

One-Sided Full Rationality

The assumption that the two sides of the market are symmetric in their level of strategic sophistication is reasonable in the context of a marriage market. However, it makes sense to think that, in other contexts, agents on different sides on the market are different in this respect. For instance, in the context of job–search, employers engage in the market more frequently than job-seekers and can gather finer information, which may lead to a better understanding of the market.

As a rough approximation of this idea, we now assume that one side of the market (women) consists of fully rational agents while the agents on the other side of the market (men) are partially cursed. A natural question is whether the existence of the fully rational agents on one side of the market alleviates the problem of oversearch or not. The next result shows that not only is the answer to this question negative, but, in fact, the share of eternal singles can be greater in this case.

Proposition 5 *Suppose that $s_v > v$ for some $v < \overline{v}$. In equilibrium, all women whose pizzazz is lower than v never marry.*

Fully rational women react to the overselectiveness of men by lowering their own standards: if men of their own caliber reject them, they turn to lower-pizzazz men. The women's lower standards raise the standards of lower-pizzazz men, thereby exacerbating the problem: the low-pizzazz men become even more selective as they are accepted by higher-pizzazz rational women and refuse to marry these women as well. This process leads to unraveling as, eventually, higher-pizzazz women will accept all men while even the lowest-pizzazz men will be unwilling to marry them.

Proposition 5 shows that even rational agents may find themselves forever single. In fact, fewer women marry when all women are fully rational than when women are boundedly rational, in which case women at the bottom of the pizzazz distribution do marry.

3.2 The Analogy-Based Expectation Equilibrium

In the previous section, we introduced a behavioral friction into the two-sided search framework. We assumed that agents' beliefs regarding the behavior of each individual on the other side of the market are affected by the average behavior of the entire population on that side, where the partial cursedness parameter allowed us to vary the magnitude of the agents' mistakes. We established that even a small departure from the rational expectations assumption can lead to extremely different outcomes.

In this section, we introduce a different behavioral friction. We assume that agents' beliefs regarding each individual depend on the behavior of only a subset of agents, whose pizzazz is similar that individual's pizzazz. The size of these subsets allows us to capture the magnitude of the agents' mistakes: the smaller the subsets, the smaller the departure from the conventional model. We show that regardless of the size of these subsets, when the market becomes less frictional, there are fewer matches each period and more agents remain single forever.

We use the analogy-based expectation equilibrium (ABEE) (Jehiel, 2005) to incorporate this idea into the model. In an ABEE, players bundle different contingencies into categories. When they assess the other players' behavior, they fail to distinguish between their behavior in different contingencies that belong to the same category. We adapt this concept by assuming that each agent divides the agents on the other side of the market into categories. The agent then believes that, in each category, all of the agents behave in the same manner. Specifically, each agent v believes that every agent in a category accepts her/him as a partner with a probability equal to the average probability with which v is accepted by the category's members.

To illustrate the agents' beliefs in an ABEE, consider a woman w who is accepted by men whose pizzazz is lower than the median and rejected by all other men. If w were fully rational, then she would realize that only low-pizzazz men are willing to marry her. In an ABEE, when $k = 1$, woman w thinks that all men are equally likely to accept her as a partner. Since half the men find her acceptable, she thinks that each men will accept her with probability 0.5. This case is equivalent to full cursedness (i.e., $\psi = 1$) in Section 3.1. Now, suppose that $k = 3$; that is, men are partitioned into three categories. Since all men in the bottom category accept w, she *correctly* believes that all men in this category will accept her as a partner. Similarly, she correctly expects each man in the top category to reject her. However, woman w's predictions are inaccurate with regard to men in the intermediate category, whom she expects to accept her with probability 0.5 regardless of their pizzazz.

As this example illustrates, unlike in the previous section, agents do not necessarily think that all other agents are achievable. When all agents in a specific category reject an individual, the latter correctly predicts that these agents are out of her/his league. The agents' beliefs are coarse only with respect to categories in which a fraction of the population accepts them.

Formally, we assume that the agents on each side of the market are partitioned into k adjacent cells $P_1, ..., P_k$, where each cell contains the same mass of agents.[8] We denote

[8] Our results are not sensitive to the assumption that each cell contains the same mass of agents.

$sup(P_j) := \overline{p}_j$ and $inf(P_j) := \underline{p}_j$. Every agent v believes that each $w \in P_j$ accepts v as a partner with probability β_{vj}. We say that a profile of beliefs $\beta = (\beta_{vj})_{v \in M \cup W, j \in \{1,...,k\}}$ is *consistent* with a profile of strategies σ if

$$\beta_{vj} = \frac{\int_{P_j \cap A_v(\sigma)} f(x)dx}{\int_{P_j} f(x)dx}$$

for every $v \in M \cup W$ and every cell $j \in \{1, ..., k\}$.

Definition 3 *A profile of strategies σ and a profile of beliefs β form an ABEE if β is consistent with σ and, for every $v \in M \cup W$, σ_v is a best response to $(\beta_{vj})_{j \in \{1,...,k\}}$.*

The next lemma establishes that, in an ABEE, agents with higher pizzazz have higher standards. This result is Lemma 1's counterpart.

Lemma 4 *In a symmetric ABEE, $\hat{a}_v$ and a_v are weakly increasing in v.*

Lemma 4 implies that agents whose pizzazz is lower than a_v reject agent v. Since, by definition, agents whose pizzazz is greater than a_v reject v, there exists at most one cell in which different cell members treat v differently. Specifically, if $a_v \notin P_j$, then either all of the cell's members reject v or all of them accept v. Either way, v correctly predicts the cell's members' behavior. On the other hand, if $a_v \in P_j$, then agent v's beliefs regarding that cell are coarse.[9] Hence, each agent v holds accurate beliefs regarding the behavior of agents in at least $k - 1$ cells and inaccurate beliefs regarding the agents in at most one of the cells. The larger k is, the larger the share of the population about whom the agents' estimates are accurate. Thus, k is a measure of the agents' mistakes. The next corollary formalizes this discussion.

Corollary 3 *In a symmetric ABEE, agent v correctly assesses the probability with which agent $w \in int(P_j)$ accepts her/him as a partner if and only if $a_v \notin int(P_j)$.*

Once monotonocity is established, Lemma 2 holds as its proof is independent of the behavioral model (it relies only on the cutoffs being monotone). Hence, $s_v < v$ is both necessary and sufficient for marriage in equilibrium. We now derive the formula for s_v, which does depend on the behavioral model.

[9]If $a_v = \underline{p}_j$ or $a_v = \overline{p}_j$, then agent v correctly predicts the behavior of all agents with the possible exception of agent a_v.

An agent who uses an acceptance cutoff $v \in P_j$ and whose opportunity value is v rejects all agents who belong to lower cells and, by Corollary 3, expects agents in higher cells to reject her/him. Thus, our agent understands that mutual acceptance is only possible when meeting agents who belong to P_j.

Let $v \in P_j$. The share of agents in P_j that an agent with an acceptance cutoff v accepts is $k(F(\overline{p}_j) - F(v))$. The probability of meeting a member of P_j is $\frac{\mu}{k}$. With an opportunity value of v, the agent expects to be accepted by each member of P_j with probability $k(F(v) - F(\underline{p}_j))$. Thus, an agent with an acceptance cutoff v and an opportunity value v expects mutual acceptance with probability $\mu k(F(v) - F(\underline{p}_j))(F(\overline{p}_j) - F(v))$ and, conditional on marriage, an expected payoff of $E[w|v < w < \overline{p}_j]$. Hence, for every $v \in P_j$,

$$s_v = \frac{\delta\mu k(F(v) - F(\underline{p}_j))(F(\overline{p}_j) - F(v))E[w|v < w < \overline{p}_j]}{1 - \delta(1 - \mu k(F(v) - F(\underline{p}_j))(F(\overline{p}_j) - F(v)))} \tag{7}$$

Note that $s_{\underline{p}_j} = 0 = s_{\overline{p}_j}$ and that (7) is continuous within each category. Thus, the condition for marriage is always satisfied at the boundaries of each cell, and, in particular, at $\overline{v}$ and $\underline{v}$. Hence, only agents with medium pizzazz may remain single forever.

In the next result, we use (7) and Lemma 2 to obtain comparative statics. As in the previous section, when the market becomes less frictional, the share of agents who search indefinitely increases and, when search frictions vanish, the market collapses.

Proposition 6 *The share of agents who marry in a symmetric ABEE weakly decreases in μ and δ and converges to* 0 *when δ goes to* 1.

This result shows that the market collapses as search frictions vanish even when agents do not expect to marry agents who are much more desirable than themselves. This is because, when search frictions are sufficiently small, a_v and v belong to the same cell. Thus, agents misestimate the probability with which they are accepted only with regard to agents who belong to the same cell as themselves. Hence, in an ABEE, when search frictions vanish, the agents' expectations are realistic in the sense that they do not expect to marry agents who are out of their league.

ABEEs can be constructed in a similar way to partially cursed equilibria. As the proof of existence is similar to the proof of Proposition 4, it is omitted.

4 Concluding Remarks

We studied a model of the marriage market in which the participants' reasoning is coarse. In equilibrium, individuals who are more desirable have higher standards and agents vary in their perception of this correlation. Agents who underestimate it overvalue their prospects in the market as they put too much weight on the possibility of marrying highly attractive individuals. As a result, they set standards that are too high and search longer than is optimal. In equilibrium, this leads to prolonged singlehood and may even result in an eternal search. Our results imply that when agents are not fully rational, technological advances that thicken markets and enable faster and more efficient search can exacerbate the agents' biases and make them overall worse off.

Throughout the analysis we assumed that agents who marry obtain the pizzazz of their spouse or, in Burdett and Coles' (1997) words, "Looking in the mirror to admire one's own pizzazz does not increase utility." While this is natural in some contexts, in others there is some complementarity between partners. The main results and intuitions of the paper hold in many of these settings (e.g., when the payoff function is multiplicatively separable, as analyzed in Eeckhout, 1999). In fact, as long as agents with higher pizzazz have higher standards, which is necessary in any form of assortative mating, our qualitative results hold.

Although we use the marriage terminology in this paper, we wish to stress that the model and the main insights have implications for the labor market as well. As in the marriage market, new search technologies have changed the way people search for a job. For example, social networks such as LinkedIn enable employers and job-seekers to match faster than ever before. In the context of job search, additional factors may come into play as employers and potential hires can negotiate wages. However, as long as utility is not fully transferable and the job-seekers' preferences over employers are correlated, there will be some vertical heterogeneity and our insights will remain valid. In particular, as the market becomes less frictional, it can be more difficult for job-seekers and employers to match.

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5 Appendix: Proofs

Proof of Lemma 1. Let $v < w$. Since agents use cutoff strategies, it follows that $A_v \subseteq A_w$. Hence, $a_v \leq a_w$. Moreover, $A_v \subseteq A_w$ also implies that $\gamma_v(\cdot) \leq \gamma_w(\cdot)$. Hence, if agent w uses agent v's optimal acceptance cutoff $\hat{a}_v$, then w will obtain a (perceived) expected payoff of at least $U_v^\star$. Therefore, $U_w^\star \geq U_v^\star$ and, hence, $\hat{a}_w \geq \hat{a}_v$.

Proof of Lemma 2. In a symmetric equilibrium, if agent v rejects matches with agents of the opposite sex whose pizzazz is v, then v is rejected by agents with pizzazz v on the other side of the market as well. That is, $\hat{a}_v > v$ implies that $a_v < v$. By the same logic, $\hat{a}_v = v$ implies that $a_v \geq v$ and $\hat{a}_v < v$ implies that $a_v \geq v$. Note that agent v marries in finite time if and only if $\hat{a}_v < a_v$. Thus, agent v marries in finite time if and only if $\hat{a}_v \leq v \leq a_v$, with at least one strict inequality.

Note that, given an opportunity value of $a_v = v$, an agent can obtain a (perceived) discounted expected payoff of s_v by using an acceptance cutoff of v, but this is not necessarily optimal. That is, if $a_v = v$, then $\delta U_v^\star \geq s_v$. As $U_v^\star$ is strictly increasing in a_v, it follows that $a_v > v$ implies that $\delta U_v^\star > s_v$. Since $\hat{a}_v = max\,\{\underline{v}, \delta U_v^\star\}$, $a_v = v$ implies that $\hat{a}_v \geq s_v$ and $a_v > v$ implies that $\hat{a}_v > s_v$.

In order to show necessity, let $s_v \geq v$ and split the analysis into three cases: $a_v < v$, $a_v > v$, and $a_v = v$. First, note that if $a_v < v$, the necessary condition for marriage

$\hat{a}_v \leq v \leq a_v$ is violated. Second, let $a_v > v$. As we showed in the previous paragraph, $a_v > v$ implies that $\hat{a}_v > s_v$. Since $s_v \geq v$, we obtain a contradiction to the necessary condition for marriage $\hat{a}_v \leq v \leq a_v$. Third, let $a_v = v$. As we showed in the previous paragraph, $a_v = v$ implies that $\hat{a}_v \geq s_v$. Since $s_v \geq v$ and $a_v = v$, it follows that $\hat{a}_v \geq a_v$, which contradicts the necessary condition for marriage $a_v > \hat{a}_v$.

In order to show sufficiency, let $s_v < v$. We start by proving the result under the assumption that $v > \underline{v}$ and then take care of the case where $v = \underline{v}$. As before, we split the analysis into three cases: $a_v < v$, $a_v = v$, and $a_v > v$. First, let $a_v > v$. Symmetry implies that $\hat{a}_v \leq v$ such that the sufficient condition for marriage $a_v > \hat{a}_v$ holds.

Second, let $a_v = v$. We now show that, if $s_v < v$ and $v > \underline{v}$, then $a_v = v$ implies that $\hat{a}_v < v$ such that the sufficient condition for marriage $\hat{a}_v < a_v$ holds. Due to symmetry, it cannot be that $a_v = v$ and $\hat{a}_v > v$ both hold. We now show that it is also impossible that $a_v = v$ and $\hat{a}_v = v$ both hold. To see why, note that the optimality of an acceptance cutoff $\hat{a}_v = v$ given $a_v = v$ implies that $s_v = \delta U_v^\star$ in this case. Since $s_v < v$ and $\hat{a}_v = v$, it follows that $\hat{a}_v > \delta U_v^\star$. As $\hat{a}_v = max\{\underline{v}, \delta U_v^\star\}$, it follows that $\hat{a}_v = \underline{v}$. Since $\hat{a}_v = v$, we obtain a contradiction to the assumption that $v > \underline{v}$. In conclusion, if $a_v = v$, it must be that $\hat{a}_v < v$.

Third, let $a_v < v$. Symmetry implies that $\hat{a}_v > v$. As $\hat{a}_v > v$, it follows that $\delta U_v^\star > v$. As $U_v^\star$ is increasing in a_v, it follows that increasing the opportunity value to $a_v = v$ does not change the inequality $\delta U_v^\star > v$. Thus, an opportunity value of $a_v = v$ implies an acceptance cutoff $\hat{a}_v > v$, in contradiction to the fact that, in a symmetric equilibrium, $a_v = v$ implies that $\hat{a}_v \leq v$. In conclusion, if $s_v < v$ and $v > \underline{v}$, then the sufficient condition for marriage holds as either $a_v > v \geq \hat{a}_v$ or $a_v = v > \hat{a}_v$.

In order to complete the proof, we show that if $s_{\underline{v}} < \underline{v}$, then agent $\underline{v}$ marries in finite time. Assume to the contrary that $a_{\underline{v}} = \underline{v}$. Thus, $\hat{a}_v > \underline{v}$ and $\delta U_v^\star > \underline{v}$ for every $v > \underline{v}$. Denote by z a number such that $a_v = z$ induces $\delta U_v^\star = \underline{v}$. Since $\underline{v} > 0$, it follows that $z > \underline{v}$. Choose $w \in (\underline{v}, z)$ and note that for any $v \in (\underline{v}, \hat{a}_w)$, it holds that $a_v \leq w < z$. Thus, for any such v, $\delta U_v^\star < \underline{v}$, in contradiction to $\hat{a}_v > \underline{v}$ being part of an equilibrium. We can conclude that $a_{\underline{v}} \neq \underline{v}$. Thus, $a_{\underline{v}} > \underline{v}$. By symmetry, $\hat{a}_{\underline{v}} = \underline{v}$ and the sufficient condition for marriage $a_{\underline{v}} > \hat{a}_{\underline{v}}$ holds.

Proof of Proposition 2. From (5), we can see that s_v is strictly increasing in δ, μ, and ψ. Moreover, at the $\delta = 1$ limit, s_v converges to $E[w|w \geq v] > v$ for all $v \in (\underline{v}, \overline{v})$. Thus, the share of agents who satisfy Condition 6 decreases in μ, ψ, and δ, and converges to 0 when δ goes to 1.

Proof of Proposition 3. This proof consists of three steps. First, we show the existence of a threshold $v^1 < \overline{v}$ such that agents whose pizzazz is higher than v^1 behave as if they were rational. Second, we establish the existence of a threshold $v^2 > \underline{v}$ such that $\hat{a}_v = \underline{v}$ for every $v < v^2$. Lastly, we show that agents whose pizzazz is lower than v^1 overvalue the prospect of remaining single and, if their pizzazz is also greater than v^2, they oversearch.

Due to the search frictions, the agents' continuation values are bounded by $\delta\overline{v}$. Hence, in equilibrium, agents whose pizzazz is greater than $\delta\overline{v}$ are accepted by all agents of the opposite sex. Thus, $a_{\delta\overline{v}} = \overline{v}$. By Lemma 1, in equilibrium, there exists pizzazz $v^1 < \overline{v}$ such that $a_v = \overline{v}$ for $v \geq v^1$ and $a_v < \overline{v}$ for $v < v^1$. Since $\sigma_w(v) = 1$ for every $v \geq v^1$ and any w, it follows that $\gamma_v(w) = 1$ for any $v \geq v^1$ and any w. Thus, every agent $v \geq v^1$ forms correct expectations and, therefore, behaves as if (s)he were fully rational. Note that an acceptance cutoff of v^1 is optimal given a belief that one is accepted by every agent of the opposite sex both under partial cursedness and under the rational expectations model. Thus, $[v^1, \overline{v}]$ is the top class in Proposition 1.

In order to establish the threshold v^2, note that $s_{\underline{v}} = 0 < \underline{v}$. Thus, agents at the bottom of the distribution marry in equilibrium. Hence, $\hat{a}_v = \underline{v}$ for some $v > \underline{v}$. By Lemma 1, there exists $v^2 > \underline{v}$ such that $\hat{a}_v = \underline{v}$ for every $v < v^2$.

Next, we show that agents whose pizzazz is $v \in [v^2, v^1)$ oversearch and that agents whose pizzazz is lower than v^2 behave as if they were fully rational.

First, suppose that $a_v > \hat{a}_v$. Consider agent v's perceived probability of marriage. In each period, agent v correctly believes that (s)he will accept a match with probability $\mu(1 - F(\hat{a}_v))$. Agent v also believes that, conditional on accepting the match, (s)he will be accepted with probability

$$\psi F(a_v) + (1 - \psi)\frac{F(a_v) - F(\hat{a}_v)}{1 - F(\hat{a}_v)}. \tag{8}$$

However, conditional on accepting the match, v is accepted with probability $\frac{F(a_v) - F(\hat{a}_v)}{1 - F(\hat{a}_v)}$, which is smaller than (8) unless $\hat{a}_v = \underline{v}$ or $a_v = \overline{v}$ (in both of these cases, the two expressions are equal and agent v correctly estimates this probability). Thus, if $v \in [v^2, v^1)$, then (s)he underestimates the time it will take her/him to marry.

Now consider the perceived expected pizzazz of agent v's partner in a potential marriage. Agent v will marry an agent whose expected pizzazz is $E[w|\hat{a}_v \leq w < a_v]$. However, v believes that (s)he will marry an agent whose expected pizzazz is

$$\frac{\psi F(a_v)(1-F(\hat{a}_v))E[w|\hat{a}_v \leq w] + (1-\psi)(F(a_v)-F(\hat{a}_v))E[w|\hat{a}_v \leq w < a_v]}{\psi F(a_v)(1-F(\hat{a}_v)) + (1-\psi)(F(a_v)-F(\hat{a}_v))},$$

which is higher than $E[w|\hat{a}_v \leq w < a_v]$, unless $a_v = \overline{v}$, in which case the two expressions are equal. Thus, if $v < v^1$, then (s)he overestimates the expected pizzazz of her/his eventual partner.

In conclusion, unless $a_v = \overline{v}$, in which case v is correct, agent v's perceived discounted expected payoff, $U_v^\star$, is higher than the actual discounted expected payoff v obtains. Thus, whenever the agent chooses an acceptance cutoff $\hat{a}_v > \underline{v}$, it is too high as well (since $\hat{a}_v = \delta U_v^\star$). It follows that every $v \in [v^2, v^1)$ searches longer than optimal given a_v. Finally, if setting a cutoff $\hat{a}_v = \underline{v}$ is optimal given v's perceived expected payoff, then it is also optimal given the correct expected payoff, which is weakly lower. Hence, every agent whose pizzazz is $v \leq v^2$ behaves as if (s)he were fully rational.

To complete this part of the proof, consider the case where $\hat{a}_v \geq a_v$. Since $s_{\underline{v}} = 0 < \underline{v}$, agent $\underline{v}$ must marry and, therefore, $a_v = \hat{a}_v = \underline{v}$ cannot hold in equilibrium. It follows that $a_{\underline{v}} > \underline{v}$ and, by monotonicity, $a_v > \underline{v}$ for all $v > \underline{v}$. Thus, agent v can marry by setting a low cutoff. However, when $\hat{a}_v \geq a_v$, agent v marries with probability 0 and gains an actual expected payoff of 0. Thus, the agent's acceptance cutoff is higher than optimal and the agent searches longer than optimal given a_v.

Proof of Lemma 3. Assume that C is a marriage class and $\overline{c} \neq \overline{v}$. By the definition of a class, $\hat{a}_v \geq \overline{c}$ for any agent $v > \overline{c}$. Since C is in a marriage interval, $\lim_{v \to \overline{c}^+} \hat{a}_v = \overline{c}$. There exists a unique pizzazz $a_{\overline{c}} > \overline{c}$ such that an acceptance cutoff of $\overline{c}$ is optimal given an opportunity value $a_{\overline{c}}$. Thus, $a_{\overline{c}} = \lim_{v \to \overline{c}^+} a_v$, and, as a result, $\hat{a}_{a_{\overline{c}}} = \overline{c}$. By monotonicity, $\hat{a}_v = \overline{c}$ for any $v \in (\overline{c}, a_{\overline{c}})$. This implies that $a_v = a_{\overline{c}}$ for all such v. Therefore, $C' = [\overline{c}, a_{\overline{c}})$ is a class.

Assume that C is a singles class and $\overline{c} \neq \overline{v}$. For any $v > \overline{c}$, it holds that $a_v \geq \overline{c}$. Since C is in a singles interval, $\lim_{v \to \overline{c}^+} a_v = \overline{c}$. There exists a unique acceptance cutoff $\hat{a}_{\overline{c}} > \overline{c}$ that is optimal given an opportunity value $\overline{c}$. By monotonicity, $a_v = \overline{c}$ for any $v \in (\overline{c}, \hat{a}_{\overline{c}})$. This implies that $\hat{a}_v = \hat{a}_{\overline{c}}$ for all such v. Therefore, $C' = [\overline{c}, \hat{a}_{\overline{c}})$ is a class.

The proofs of the existence of C'' follow the same logic and are omitted for brevity.

Proof of Proposition 4. We consider each marriage/singles interval separately and define, for every agent v, an acceptance cutoff $\hat{a}_v$. First, consider singles intervals such that $s_v = v$ for any pizzazz v in the interval. For each agent in these intervals, set $\hat{a}_v = v$. In the remainder of the proof, singles intervals are assumed to contain only agents with $s_v > v$.

First, we show that, for any opportunity value $a \in L$, there exists an acceptance cutoff $\hat{a} \in L$ such that $\hat{a}$ is optimal given a. Let L be a marriage/singles interval that does not contain $\underline{v}$ or $\overline{v}$, and let $a \in L$ and $\epsilon \in \mathbb{R}$. Note that L is an open interval. Define $v = a + \epsilon$. Consider v's discounted perceived expected payoff, δU_v, as a function of ϵ, assuming that $\hat{a}_v = v$ and $a_v = a$. If L is a marriage interval, then for $\epsilon = 0$, it holds that $v = a$ and so $\delta U_v = s_v < v$. On the other hand, for $\epsilon = \underline{l} - a$, it holds that $v = \underline{l}$ and so $\delta U_v > \underline{l} = v$ ($\underline{l}$'s discounted perceived payoff when $a_{\underline{l}} = \hat{a}_{\underline{l}} = \underline{l}$ is $\underline{l}$). If L is a singles interval, then for $\epsilon = 0$, $\delta U_v = s_v > v$. For $\epsilon = \overline{l} - a$, $\delta U_v < \overline{l} = v$ ($\overline{l}$'s discounted perceived payoff given $a_{\overline{l}} = \hat{a}_{\overline{l}} = \overline{l}$ is $\overline{l}$). In either case, the payoff is continuous in ϵ and, therefore, for some ϵ, there exists $\hat{a} \in L$ such that $\delta U_{\hat{a}} = \hat{a}$.

Second, consider now the dual exercise: for a given acceptance cutoff $\hat{a} \in L$, we prove the existence of an opportunity value $a \in L$ that "rationalizes" it. Let L be a marriage/singles interval that does not contain $\underline{v}$ or $\overline{v}$, and let $w \in L$. If L is a marriage interval, then let $a \in [w, \overline{l}]$ and find a pizzazz $\hat{a}$ such that $\delta U_{\hat{a}} = \hat{a}$ given the opportunity value a as in the previous paragraph. If $a = w$, then $\hat{a} < w$. If $a = \overline{l}$ then $\hat{a} = \overline{l} > w$. By continuity, there exists an a such that $\hat{a} = w$ (i.e., a rationalizes the acceptance cutoff w). If L is a singles interval, then let $a \in [\underline{l}, w]$ and find a pizzazz $\hat{a}$ such that $\delta U_{\hat{a}} = \hat{a}$ given the opportunity value a as in the previous paragraph. If $a = w$, then $\hat{a} > w$. If $a = \underline{l}$, then $\hat{a} = \overline{l} < w$. By continuity, there exists an a such that $\hat{a} = w$ (i.e., a rationalizes the acceptance cutoff w).

We now use the insights from the first two paragraphs to cover an arbitrary marriage/singles interval with classes. Let L be a marriage/singles interval that does not contain $\underline{v}$ or $\overline{v}$, and let $c^0 \in L$. For $k = 1, 2, ...$, let c^k be the pizzazz for which $\delta U_v = c^{k-1}$ if $\hat{a}_v = c^{k-1}$ and $a_v = c^k$. For $n = 1, 2, ...$, let c^n be the pizzazz for which $\delta U_v = c^{-n}$ if $\hat{a}_v = c^{-n}$ and $a_v = c^{1-n}$. Note that both series $\{c^k\}_{k \in \mathbb{N}}$ and $\{c^n\}_{n \in \mathbb{N}}$ are bounded and monotonic, and hence converge to k^* and l^*, respectively. At each of these limits, $v \in \{k^*, l^*\}$, it must hold that $v = a_v = \hat{a}_v$. Thus, $v \in \{\underline{l}, \overline{l}\}$. For $k \in \mathbb{Z}$, define $C^k = [c^k, c^{k+1})$. Note that the sets $\{C^k\}_{k \in \mathbb{Z}}$ are disjoint and cover L. Set $\hat{a}_v = c^k$ for any $v \in [c^k, c^{k+1})$, $\hat{a}_{\underline{l}} = \underline{l}$, and $\hat{a}_{\overline{l}} = \overline{l}$.

Let L be a marriage/singles interval containing $\overline{v}$. Set $c^0 = \overline{v}$. For any $n = 1, 2, ...,$ let c^n be the pizzazz for which $\delta U_v = c^{n-1}$ if $\hat{a}_v = c^{-n}$ and $a_v = c^{1-n}$. In the case of $c^{-n} \leq \underline{v}$, set $c^{-n} = \underline{v}$ and stop the process. Define $C^n = [c^{-n}, c^{-n+1})$ for $n = 1, 2, ...$. As in the previous case, the sets $\{C^{-n}\}_{n \in \mathbb{N}}$ are disjoint and cover L. Set $\hat{a}_v = c^{-n}$ for any $v \in [c^{-n}, c^{1-n})$ and $\hat{a}_{\overline{v}} = c^{-1}$.

Let L be a marriage/singles interval containing $\underline{v}$ but not $\overline{v}$. Set $c^0 = \underline{v}$. For any $k = 1, 2, ...,$ let c^k be the pizzazz for which $\delta U_v = c^{k-1}$ if $\hat{a}_v = c^{k-1}$ and $a_v = c^k$. Define $C^{k-1} = [c^{k-1}, c^k)$ for any $k = 1, 2...$. Set $\hat{a}_v = c^{k-1}$ for any $v \in [c^{k-1}, c^k)$.

By construction, for any v for which $v = s_v$, $a_v = v$. In any other marriage/singles interval L, if $\hat{a}_v = c^k$ for some k, then $a_v = c^{k+1}$. Furthermore, all acceptance cutoffs $\hat{a}_v$ are optimal given their respective opportunity values a_v. Thus, an equilibrium exists. The existence of a cutoff $\overline{\delta}$ is implied by Proposition 2.

Proof of Proposition 5. Observe that Lemma 1 still holds (for both sides of the market). If woman v accepts man $\tilde{v}$, then every woman whose pizzazz is lower than v accepts man $\tilde{v}$ and so $a_{\tilde{v}} \geq v$. By the definition of s_v, if $a_{\tilde{v}} = v$, then $\delta U_{\tilde{v}} \geq s_v$. Since $\delta U_{\tilde{v}}$ is increasing in $a_{\tilde{v}}$, it follows that, given $a_{\tilde{v}} \geq v$, it must be that $\delta U_{\tilde{v}} \geq s_v$. Thus, $\delta U_{\tilde{v}} \geq s_v > v$. Hence, man $\tilde{v}$ rejects woman v. It follows that woman v cannot marry in equilibrium. Hence, she accepts every man she encounters. It follows that no man ever accepts woman v in equilibrium. By monotonicity, no man ever accepts a woman whose pizzazz is lower than v.

Proof of Proposition 6. Consider (7) and observe that s_v is increasing in δ and μ. Moreover, when δ goes to 1, s_v goes to $E[w|v \leq w \leq \overline{p}_j] > v$ for $v \in int(P_j)$. Thus, the share of agents for whom the necessary and sufficient condition for marriage is satisfied becomes smaller when δ and μ increase and it converges to 0 when δ goes to 1.