Multilateral Contracting with Manipulation*

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March 28, 2019

Abstract

We study multilateral reallocation of risk when the state of nature is unverifiable, such that contracts are conditioned on a state-dependent signal (e.g., net earnings in a financial report). A subset of the agents can manipulate the signal's realisation at some cost and as a result Pareto-optimal reallocation of risk is precluded. The agents can write additional bilateral side-contracts that can be used to incentivise one of the parties to manipulate the signal. Using a novel pairwise stability notion that takes into account agents' beliefs about contemporaneous deviations initiated by their counterparties, we explore the limits of risk-sharing and risk-bearing.

1 Introduction

It is well known that when economic agents have access to Arrow–Debreu securities, they can reallocate risk efficiently. In practice, however, state-contingent contracts are not always feasible, as the state of nature may be unobservable, unverifiable, or hard to assess to the point where state-contingent contracts are unenforceable or too costly to implement.

For these reasons, risk is often reallocated by means of contracts that are contingent on verifiable variables that are informative about the state of nature. For example, as Duffie and Stein (2015) pointed out, financial benchmarks such as the inter-bank offered rates "have been heavily used in contracts whose purpose is to transfer risk related

*A previous version of this paper was circulated under the title "Multilateral Risk-Sharing with Manipulation." This paper has benefited from ERC grant 230251. I am particularly grateful to Rani Spiegler for many insightful discussions and useful suggestions. I thank Benjamin Bachi, Daniel Bird, Eddie Dekel, Kfir Eliaz, Alex Frug, Jacob Glazer, Santiago Oliveros, Debraj Ray, Ariel Rubinstein, Joel Sobel, and Asher Wolinsky for helpful comments and suggestions.

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to fluctuations in general market-wide interest rates." Other prominent examples are insurance contracts, which are often contingent on an appraisal rather than on actual damage, and managerial compensation contracts, which are often contingent on a firm's net earnings as they appear in its financial reports rather than contingent on the firm's actual performance. In the present paper, we focus on contracts of this kind and refer to the contractible variable as a *signal* about the state of nature.

Transferring risk in these types of contracts gives rise to a *moral hazard* problem that results from the agents' ability to *manipulate* the signal's realisation by taking costly actions. Such costly actions include: forging an appraisal or misreporting the occurrence of an insurable event; deferring recognition of some expenditure to change a firm's net earnings at a specific date; inflating future prices in a commodity market by placing large buy orders in the underlying market; and hiring lobbyists to influence a governor's declaration on which a contract depends.

We study reallocation of risk among $n > 2$ agents. Risk is reallocated by means of contracts, which are balanced-budget transfers contingent on a signal that reveals the state of nature. We refer to the collection of these contracts as the *multilateral contract*. Some of the agents have the ability to manipulate the signal's realisation unilaterally by incurring some cost. When for each agent the cost of manipulating the signal is greater than the corresponding benefit, the multilateral contract is said to be incentive compatible (IC), and the signal *perfectly reveals* the state of nature (i.e., there is no manipulation). To illustrate some of the model's features, we present the following example.

Example 1 *There are two states, high (H) and low (L). Alice and Bob are each exposed to a negative shock of* 100 *dollars in state L. There is a risk-neutral insurer who is willing to share some of the risk for a premium. Risk-sharing contracts are contingent on an appraisal $s \in \{h, l\}$ made by a certified appraiser. The appraiser's report is h in state H and l in state L. Alice and Bob both know the appraiser. In state H, each of them can pay the appraiser a bribe of* 90 *dollars so that he will change his appraisal from h to l. Observe that full insurance is not IC as it incentivises the agents to bribe the appraiser in state H. Because of the moral hazard, each of them can receive a coverage of at most* 90 *dollars. We shall refer to such insurance contracts as constrained-efficient contracts.*

A key feature in this work is that, at the contracting stage, before the state is realised, agents can add new bilateral contracts to the existing multilateral contract without withdrawing from it. We refer to these contracts as *side-contracts*. The purpose of an additional side-contract can be to provide legitimate mutual insurance or to incentivise one of the contracting counterparties to manipulate the signal. Side-contracts that are signed with the intention that one of the counterparties will manipulate the signal introduce a new source of instability into multilateral reallocation of risk since they impose an *externality on third parties*.

Using Example 1, let us demonstrate how a pair of agents may benefit from the addition of a side-contract that incentivises one of them to manipulate the signal. Consider a multilateral contract in which Alice and Bob each receive a coverage of 90 dollars. Recall that this is the maximal coverage that each of them can obtain in an IC multilateral contract. Both Alice and Bob are better off if, at the contracting stage, they add a side-contract in which Bob pays Alice a small $\epsilon > 0$ if and only if $s = l$. This side-contract violates Alice's incentive-compatibility constraint and incentivises her to bribe the appraiser in state H, which makes Bob better off as he guarantees his preferred appraisal by paying a small cost of ϵ. The contract between Alice and Bob violates the stability of the multilateral contract because it makes both agents better off when the possibility of ex-post manipulation is taken into account.

An insurer who predicts the side-contract between Alice and Bob will be unwilling to provide them with coverage since it exposes him to a *negative externality* imposed on him by the ex-post manipulation of the appraisal. Bilateral side-contracts may have a negative effect on a third party due to the contracting parties' ability to manipulate the contractible variable ex post. This *contractual externality* plays a key role in our model.

Our primary objective is to study the implications of potential manipulations on the ability to reallocate risk in multilateral environments. To refrain from making strong assumptions about the contracting process, we take a "cooperative" approach in the spirit of the network formation literature (see Jackson and Wolinsky, 1996): a multilateral contract is said to be *pairwise stable* if no pair of agents is better off adding a new side-contract (without withdrawing from the existing multilateral contract). The multilateral contract can be interpreted as the sum of a *network of bilateral contracts*, and a deviation from the contract can be interpreted as an addition of a bilateral link to the prevailing web of contracts.

We show that an IC pairwise stable multilateral contract does not exist in various configurations of interest. This result follows from the fact that pairwise stability considers one deviation at a time, which is an implicit assumption that each agent i who takes part in some deviation believes that there are no additional deviations that make him worse off if he agrees to take part in the deviation. In particular, pairwise stability assumes that i believes that his counterparty to the deviation does not have an *ulterior motive* such as another side-contract (with another agent) that is not observed by i. This assumption is especially restrictive in a multilateral setting as the benefit of a side-contract is affected by ex-post manipulations by third parties.

We relax the assumption mentioned above by developing a new weaker pairwise stability notion, which we shall refer to as *weak stability.* Weak stability incorporates considerations from the Nash equilibrium refinements literature into a concept of stability in the spirit of cooperative game theory. The idea behind weak stability is that each pairwise deviation can be viewed as if it were initiated by one of the deviating parties, say, i. An agent j who receives an offer to take part in this deviation conjectures what other deviation agent i may have initiated with another agent since it has an effect on the attractiveness of i's offer. The only restriction we impose on agent j's conjecture is that it must *rationalise the observed offer.* That is, according to j's conjecture, i's offer to j makes i better off. We refer to such a conjecture as a *consistent* conjecture. Agent j rejects i's offer if there exists a consistent conjectured deviation that makes j worse off if he agrees to i's offer.

Our main result is that, under mild domain restrictions, weakly stable contracts are not constrained-efficient. We show that weakly stable contracts exist, but that the contractual externalities constrain the agents' ability to transfer risk. It is worth pointing out that weak stability is defined by using conservative restrictions on the deviating agents' beliefs. If instead we were to use a stronger set of restrictions on the deviating agents' beliefs, then the amount of risk that could be transferred in a stable contract would be even lower.

We present two applications of the model in which we examine the implications of contractual externalities on the *volume* of risk-sharing. In the first application, we study a reinsurance market in which external reinsurers provide coverage to primary insurers who are exposed to an aggregate shock.[1] We assume that only some of the local insurers can manipulate the contractible variable and interpret the share of manipulators as a proxy for the level of corruption in the economy. For example, a high proportion of manipulators corresponds to an economy in which "revolving doors" between the public and private sectors are widespread. We derive a closed-form solution to the maximal level of risk-sharing that can be sustained by means of a weakly stable contract and show that it can be significantly lower than the constrained-efficient level

[1]Reinsurance instruments (e.g., catastrophe bonds) are used to transfer risk resulting from high-volume events from insurers to reinsurers or the capital market, and their triggers are often conditioned on state-dependent signals in order to avoid moral hazard in underwriting and claim settlements (see Doherty, 1997).

of risk-sharing.

In the second application we study speculative trade among risk-neutral speculators. Under an assumption that the economy is composed of two equally sized groups of optimistic and pessimistic agents, we derive a closed-form solution to the maximal volume of speculative trade that can be sustained by means of a weakly stable contract and show that it is increasing when the agents' prior beliefs become more *polarised*. This is different from the case of bilateral speculative trade, in which the *magnitude* of the difference between the agents' beliefs has no effect on the volume of speculative trade due to the absence of contractual externalities.

In both applications, the main message is that the maximal level of risk-sharing (or risk-bearing) is U-shaped in the share of agents who can manipulate the contractible variable. That is, when corruption becomes more widespread in the economy, its effect on the maximal volume of trade is nonmonotone.

Related literature

This article is related to the risk-sharing networks literature. Bramoullé and Kranton (2007a, 2007b) study risk-sharing network formation models in which agents mitigate risk by sharing their holdings with linked partners. In these models, the agents trade off between costly link formation and better risk-sharing. Bloch, Genicot, and Ray (2008) and Ambrus, Mobius, and Szeidl (2014) consider moral hazard in risk-sharing networks. In these models, ex post, agents who are supposed to make a transfer may deviate and refuse to do so. An agent who deviates loses some of his risk-sharing links. Bloch, Genicot, and Ray (2008) characterise stable risk-sharing networks while Ambrus, Mobius, and Szeidl (2014) study the extent and structure of risk-sharing.

Laffont and Martimort (1997, 2000) develop a framework that incorporates collusion-proofness into mechanism design. In these models (as well as in Che and Kim, 2006), a fictitious third party coordinates the side-contracts between the colluding agents. Earlier work on this topic focuses on the Vickrey–Clarke–Groves mechanism's vulnerability to collusion (Green and Laffont, 1979; Crémer, 1996). Bierbrauer and Hellwig (2016) show that coalition-proof mechanisms for public good provision that satisfy a robustness condition must take the form of a voting mechanism. The implications of potential collusion have also been studied in the contexts of organisations (Tirole, 1986, 1992; Baliga and Sjöström, 1998; Mookherjee and Tsumagari, 2004; Celik, 2009) and auctions (Graham and Marshall, 1987; Jehiel and Caillaud, 1998; Marshall and Marx, 2007).

Eliaz and Spiegler (2007, 2008, 2009) take a mechanism design approach to problems in which agents are motivated to bet on the state of nature due to differences in their prior beliefs. In these models, the state is not verifiable and the agents can manipulate the contractible variable (an action or a profile of actions) by incurring some cost. The agents' ability to manipulate this variable creates incentive constraints that restrict the betting stakes. In Kahn and Mookherjee (1998), an insuree who is exposed to a private shock can purchase coverage from multiple insurers, where insurance contracts are negotiated sequentially according to an exogenously given protocol. Since there is no exclusive dealership and overinsurance may affect the insuree's incentives to exert effort (which in turn affects the contracts' outcomes), some insurers may be reluctant to provide the insuree with coverage.

Weak stability is related to farsighted stability notions (see, e.g., Harsanyi, 1974; Chwe, 1994; Ray and Vohra, 2015) that characterise outcomes that are immune to deviations by players who recognise that their own deviations may trigger a chain of deviations by other players. In particular, in the context of network formation, pairwise farsighted stability notions (e.g., Herings et al., 2009; Herings et al., 2019) have been used to extend Jackson and Wolinsky's (1996) notion of pairwise stability.[2]

Farsighted stability differs from weak stability in several aspects. First, under farsighted stability, deviations are deterred by potential *future* deviations. Second, under farsighted stability, the identity of the agent who initiates the deviation has no effect on the other agents' beliefs. Third, farsighted pairwise stability notions typically assume that deviations are observable to agents who are not part of the deviating coalition (e.g., they allow a deviation by a pair of agents (i, j) to trigger an additional deviation by a pair of agents $k \notin \{i, j\}$ and $l \notin \{i, j\}$). Finally, pairwise farsighted stability notions typically focus on pure network formation games.

Weak stability is also related to the Nash equilibrium refinements literature. Cho and Kreps (1987) provide a criterion for examining the stability of equilibria in signaling games based on forward-induction reasoning. Our stability concept employs a similar logic to coarsen the set of pairwise stable contracts. Pomatto (2018) has applied forward-induction considerations to deviations in a two-sided matching problem. He considers given allocations, and models a noncooperative deviation game in which players use forward-induction reasoning to interpret other players behaviour off the equilibrium path (all deviations are off the equilibrium path).

The paper proceeds as follows. We present the model in Section 2 and analyse it

[2]Page et al. (2005) offer a different approach (that allows for deviations by coalitions of any size) to farsighted stability in networks.

in Section 3. In Section 4 we present two applications. In Section 5 we discuss some possible extensions. Section 6 concludes.

2 The Model

There is a set of agents $I = \{1, ..., n\}$, $n > 2$, and a set of states $\Theta = \{L, H\}$. Each agent i's preferences over monetary outcomes are represented by the concave vNM function $u_i : \mathbb{R} \to \mathbb{R}$. For each $i \in I$ let π_i denote the probability that agent i assigns to the event that state H will be realised. We use $w_i(\theta)$ to denote i's endowment when the state of nature is $\theta \in \Theta$. Let $S = \{l, h\}$ be a set of signals. We use s to denote a typical element of S and θ to denote a typical element of Θ. The signal perfectly reveals the state of nature unless it is manipulated by one (or more) of the agents. The term "manipulation" will be clarified soon.

Contracts. State-contingent contracts are not available. The agents can write contracts contingent on the realised signal. A multilateral contract $g^K : S \to \mathbb{R}^n$ sets budget-balanced transfers within a group of agents $K \subseteq I$ as a function of the realised signal. We use $g_i^K(s')$ to denote the transfer that agent $i \in I$ receives according to g^K when $s = s'$, where $g_i^K(s') := 0$ for each $i \notin K$. For any two contracts $g^K, \bar{g}^{K'}$, we use $g^K + \bar{g}^{K'}$ to denote the summation of the transfers in the two contracts. That is, for each $s \in \{h, l\}$, $\left(g^K + \bar{g}^{K'}\right)_i(s) := g_i^K(s) + \bar{g}_i^{K'}(s)$. We focus on one multilateral contract that sums all the contracts signed by the agents, and we denote this contract by g. Denote the set of such multilateral contracts by G. It is useful to use a different notation for bilateral contracts. A bilateral contract $b_{ij} : S \to \mathbb{R}$ between i and j sets a transfer $b_{ij}(s)$ from j to i contingent on the signal's realisation. Let $b_{ij} + b_{kl}$ denote a contract that sums the transfers made in the two bilateral contracts b_{ij} and b_{kl}.

Manipulation. After the state is realised, each agent $i \in M \subseteq I$ can unilaterally change the signal's realisation from $s \in \{h, l\}$ to $s' \neq s$ by paying a cost of $c > 0$. We assume that the cost of manipulation is symmetric both across signals (i.e., the cost of changing the signal from h to l equals the cost of changing the signal from l to h) and across agents (i.e., all members of M incur the same cost of manipulation). We discuss the relaxation of the symmetry assumption in Section 5. For each $g \in G$, let $PM(g) = \{m \in M : |g_m(h) - g_m(l)| > c\}$ be the set of potential manipulators.

A multilateral contract g is said to be *incentive compatible* (IC) if $PM(g) = \emptyset$. When g is IC there is no manipulation such that $s = h$ if and only if $\theta = H$. We make two substantive assumptions regarding the signal's realisations that result from multilateral contracts that are not IC. The first assumption is that if *all of the agents* whose incentive-compatibility constraints are violated prefer one realisation to the other, then they manipulate the signal to that realisation when necessary (e.g., when the state is H and their preferred realisation is l) and one of them incurs a cost of c. If there is no manipulation (e.g., when the state is H and their preferred realisation is h), no cost is incurred.

Assumption 1 *For every multilateral contract $g \in G$ such that $PM(g) \neq \emptyset$, if $g_m(h) > g_m(l)$ (respectively, $g_m(h) < g_m(l)$) for every $m \in PM(g)$, then $s = h$ (respectively, $s = l$). Moreover, if $s = l$ (respectively, $s = h$) and $\theta = H$ (respectively, $\theta = L$), then one member of $PM(g)$ incurs a cost of c.*

The second assumption pertains to the case in which the set of potential manipulators, $PM(g)$, includes exactly two agents, and these agents prefer different realisations of the signal. In this case, we assume that the signal is independent of the state. We also assume that the agent whose preferred realisation matches the signal incurs a cost of c. In other words, we assume that the two agents try to impose their preferred realisation in each possible state of nature and that the probability of success is independent of the state. The agent who succeeds in imposing his preferred realisation incurs a cost of c.

Assumption 2 *For each multilateral contract $g \in G$ and each pair of agents $m, m' \in M$, if $PM(g) = \{m, m'\}$ and $g_m(h) - g_m(l) > c > -c > g_{m'}(h) - g_{m'}(l)$, then $Prob\{s = h|\theta = H, g\} = Prob\{s = h|\theta = L, g\}$ and agent m (respectively, m') incurs a cost of c if and only if $s = h$ (respectively, $s = l$).*

Our approach in the present paper is to make elementary reduced-form assumptions on how the signal is set and who incurs the cost of manipulation. Alternatively, one can model the interaction at the manipulation stage as a noncooperative game. That is, one can commit to a specific game form and analyse its equilibria. Note that for a given game form, different multilateral contracts may induce very different payoff functions as in other models of pregame contracting (see, e.g., Jackson and Wilkie, 2005). Further, in the present model, since the set of manipulators can be a proper subset of the set of agents and we impose no constraint on the multilateral risk-sharing agreements, there are virtually no restrictions on the different payoff functions that may be induced by different multilateral contracts. Assumptions 1 and 2 can be supported by a Nash equilibrium in several noncooperative games; however, in most of these games there are multiple Nash equilibria.

The timeline in the model is as follows. First, the agents write signal-contingent contracts as described above. After the contracting stage, the state of nature is realised. Subsequently, there is a manipulation stage in which the members of M may try to affect the *signal*'s realisation. Finally, the agents receive transfers according to the contracts that they have signed and the signal that results from the manipulation stage.

For each $i \in I$, we use $\succ_i$ to represent i's *indirect preferences* over contracts. The indirect preferences take ex-post manipulations into account. For example, suppose that g is IC and g' is not IC: $PM(g') = \{j\}$ and $g'_j(h) - g'_j(l) > c$. For $i \in I - \{j\}$, $g' \succ_i g$ if and only if

$$\begin{aligned} & \pi_i u_i(w_i(H) + g_i(h)) + (1 - \pi_i) u_i(w_i(L) + g_i(l)) \\ < \; & \pi_i u_i(w_i(H) + g'_i(h)) + (1 - \pi_i) u_i(w_i(L) + g'_i(h)). \end{aligned}$$

Note that in the expression on the RHS, agent i receives $g'_i(h)$ in both states since the signal is manipulated by j. For $i = j$, $g' \succ_i g$ if and only if

$$\begin{aligned} & \pi_i u_i(w_i(H) + g_i(h)) + (1 - \pi_i) u_i(w_i(L) + g_i(l)) \\ < \; & \pi_i u_i(w_i(H) + g'_i(h)) + (1 - \pi_i) u_i(w_i(L) + g'_i(h) - c). \end{aligned}$$

Observe that the manipulation cost c is taken into account only in state L, when j manipulates the signal. A contract g is said to be *individually rational* (IR) if each agent $i \in I$ prefers signing it to not signing it. Following is the notion of efficiency that we use throughout the paper.

Definition 1 *A multilateral contract g is said to be constrained-efficient if it is IR, IC, and it is not Pareto-dominated by another IC multilateral contract.*

3 Analysis

Our plan for this section is as follows. First, we present a natural notion of robustness against pairwise deviations, which we shall refer to as *pairwise stability*. Second, we show that IC pairwise-stable multilateral contracts do not exist in two natural settings. These results lead us to define a weaker notion of pairwise stability, which we shall refer to as *weak stability*. Finally, we show that weakly stable multilateral contracts exist and that they are not constrained-efficient.

3.1 Pairwise stability

We are interested in multilateral contracts that are robust to pairwise deviations. We take a "cooperative" approach as it allows us to refrain from making assumptions about the process whereby the contracts are negotiated. The following notion of stability is inspired by the network formation literature (see Jackson and Wolinsky, 1996).

Definition 2 *A multilateral contract g is said to be pairwise stable if there exists no contract b_{ij} such that $g + b_{ij} \succ_i g$ and $g + b_{ij} \succ_j g$.*

Observe that in Jackson and Wolinsky's notion of pairwise stability, every two agents are either connected to each other or not. That is, the conventional definition of pairwise stability refers to binary links. Our notion of stability is slightly different from that of Jackson and Wolinsky since here bilateral contracts are vectors that specify budget-balanced transfers between contracting agents.

We illustrate the inherent instability in multilateral contracting using two natural settings: speculative trade among risk-neutral agents and risk-sharing among risk-averse agents. In the first setting, the agents trade to *increase* their exposure to the state of nature because of the difference in their prior beliefs. In the second setting, the agents trade to *reduce* their exposure to the state of nature because of their risk aversion. We show that in both of these settings, there exists no multilateral contract that is both IC and pairwise stable.

Proposition 1 *Let $n > 3$; suppose that $\pi_i \neq \pi_j$ for each pair of agents $i \neq j$, and assume that for each $i \in I$, u_i is linear. Then, there exists no multilateral contract g that is both IC and pairwise stable.*

Proposition 1 establishes that trade motivated purely by different prior beliefs must result in a multilateral arrangement that is either not IC or not pairwise stable. The proof shows that if the multilateral contract is not constrained-efficient, then there are two agents who are better off writing a side-contract that increases their exposure to the state. If the multilateral contract is constrained-efficient, then the stakes of the contract are high (i.e., the agents' exposure to the signal is high) such that there is a pair of agents who are better off writing a side-contract with the intention that one of them will manipulate the signal ex post.

Proposition 2 considers a risk-sharing economy in which the agents' primary goal is to reduce their exposure to the state of nature. We now impose two mild domain restrictions. We refer to the first restriction as *richness*. In our model, there are four possible "types" of agents: manipulators or nonmanipulators with positive or negative initial exposure to the state of nature (agent i's initial exposure is $w_i(H) - w_i(L)$). We consider an economy that contains at least one agent of each type.

Definition 3 *The economy is said to satisfy richness if there are two agents $m, m' \in M$ such that $w_m(H) - w_m(L) > 0 > w_{m'}(H) - w_{m'}(L)$ and two agents $i, i' \notin M$ such that $w_i(H) - w_i(L) > 0 > w_{i'}(H) - w_{i'}(L)$.*

Note that richness rules out purely aggregate shocks (see, e.g., Example 1). We shall relax richness and study aggregate shocks in Section 4. We refer to the second restriction as *nontriviality*. The latter restriction is an assumption that the manipulation cost is small in the sense that it is lower than the initial exposure of at least two members of M to the state of nature. Since the agents' primary goal in this case is to reduce their exposure to the state of nature, it follows that when the manipulation cost is very high with respect to the agents' initial exposure to the state, manipulation becomes irrelevant and the model collapses to a conventional risk-sharing economy.

Definition 4 *The economy is said to satisfy nontriviality if there exist two agents $m, m' \in M$ such that $w_m(H) - w_m(L) \geq c$ and $w_{m'}(L) - w_{m'}(H) \geq c$.*

Proposition 2 *For each $i \in I$, let $\pi_i = \pi \in (0, 1)$ and let u_i be strictly concave. If nontriviality and richness are satisfied, then there exists no multilateral contract g that is both IC and pairwise stable.*

The proof of Proposition 2 establishes that multilateral contracts that provide inefficient coinsurance are not pairwise stable because there is always at least one pair of agents who are better off coinsuring. The proof also shows that if the multilateral contract is constrained-efficient, then there is at least one pair of agents who are better off writing a side-contract that incentivises one of them to manipulate the signal.

Propositions 1 and 2 demonstrate the instability that is inherent in multilateral contracting. They suggest that pairwise stability, which is widely used in the contexts of matching and network formation, cannot be used to study the formation of networks of contracts where the contractible variable can be manipulated.

The notion of pairwise stability considers the addition of one contract at a time, which is an implicit assumption that when a pair of agents deviate by writing a side-contract b_{ij}, each of the deviating agents holds a belief that there is no other deviation that will make him worse off if he agrees to b_{ij}. This assumption is particularly restrictive since the attractiveness of a contract (and, in particular, of taking part in a deviation) is affected by the existence of side-contracts between other agents that may create an incentive to manipulate the signal for some of these agents. An agent i who takes part in a deviation from the existing multilateral contract may suspect that his counterparty to the deviation (who has already shown a tendency to steer away from the norm) has an ulterior motive such as an additional side-contract (with another agent) that makes i worse off if he agrees to take part in the deviation. In the next subsection, we coarsen pairwise stability by developing a weaker pairwise stability notion that relaxes this implicit assumption and takes into account suspicion of agents who initiate deviations from the multilateral contract.

3.2 Weak stability

Let us consider a possible deviation b_{ij}. It can be viewed as if it were initiated by one of the two agents, say, i. Pairwise stability includes an implicit assumption that j believes that i did not initiate any contemporaneous deviation b_{ik} that makes j worse off if he agrees to i's offer to deviate. Before we relax this assumption, we present an example in which healthy suspicion of i's motivation is relevant.

Example 2. Let $I = \{1, ..., 8\}$, $\pi_1 > ... > \pi_8$, $u_i(z) = z$ for each $i \in I$, and $M = \{1, 2, 3, 6, 7, 8\}$. The table summarises the agents' transfers in the contract g.

Agent	1	2	3	4	5	6	7	8
$g_i(h) - g_i(l)$	c	c	c	0	0	$-c$	$-c$	$-c$

We present two side-contracts that violate the pairwise stability of g. The first side-contract is a bet between agents 4 and 5. We show that when this bet is initiated by agent 4, then agent 5 has reason to suspect that agent 4 signed an additional contract with a third agent k, thereby incentivising k to manipulate the signal ex post.

Suppose that agent 4 initiates a side-contract b_{45} such that $b_{45}(h) > 0 > b_{45}(l)$. Agent 5 might suspect that agent 4 has initiated another side-contract b_{34} such that $b_{34}(h) = \varepsilon > 0 = b_{34}(l)$, that is, a deviation in which agent 4 incentivises agent 3 to manipulate the signal from l to h by paying him $\varepsilon > 0$ if and only if the realised signal is h. If ϵ is sufficiently small, then $g + b_{34} + b_{45} \succ_4 g + b_{45}$ (i.e., given b_{45}, the contract b_{34} makes agent 4 better off). Agreeing to b_{45} exposes agent 5 to a negative externality imposed by agent 3's manipulation of the signal (as a result of b_{34}). Note that, given b_{34}, agent 5 is worse off agreeing to b_{45}.

We now present a second deviation in which agents 6 and 7 write a side-contract with the intention that agent 7 will manipulate the signal from h to l in state H. When agent 6 initiates such a deviation, agent 7 may suspect that agent 6 has an ulterior motive in the form of an additional side-contract with agent 8. The conjectured contract between agents 6 and 8 incentivises agent 6 to manipulate the signal himself. Agent 7 suspects that agent 6 is using their side-contract b_{76} to make him manipulate the signal and pay the manipulation cost instead of doing so himself. That is, agent 7 suspects that agent 6 is trying to free-ride on him.

Suppose that agent 6 initiates a side-contract b_{76} such that $b_{76}(l) = \varepsilon > 0 = b_{76}(h)$. That is, agent 6 makes an offer to agent 7 that is supposed to break 7's indifference and incentivise him to manipulate the signal from h to l in state H. Agent 7 may suspect that agent 6 has also initiated a side-contract b_{68} such that $b_{68}(s) = b_{76}(s)$ for each $s \in S$. Observe that $g_6(l) - g_6(h) + b_{68}(l) - b_{68}(h) = c + \epsilon > c$. By Assumption 1, under $g + b_{68}$, the signal is $s = l$ for each $\theta \in \{H, L\}$. If ϵ is sufficiently small, then $g + b_{68} + b_{76} \succ_6 g + b_{68}$ since the manipulation cost is paid by agent 7 instead of agent 6. That is, the contract b_{76} that is observed by agent 7 can be rationalised by the conjectured contract b_{68}. Note that the realised signal is identical in both cases (i.e., whether or not agent 7 agrees to b_{76}). However, the identity of the agent who pays for the manipulation is different. Agreeing to b_{76} makes agent 7 pay the cost of manipulation instead of agent 6 paying this cost. If ε is small relative to c, agent 7 is worse off agreeing to agent 6's offer to deviate.

We now present a notion of stability that takes into account the suspicion motive presented above. This notion involves suspicion of agents who break the norm and initiate deviations from the existing multilateral contract. Given a multilateral contract g and an offer to deviate and sign a side-contract b_{ij}, made by i, $\beta_j(b_{ij}, g) \in \{b_{ik} | k \in I - \{i, j\}\} \cup \emptyset$ denotes agent j's belief about an additional contemporaneous offer that i has made to another agent. We implicitly assume that agent j cannot observe deviations that do not include him. If $g + \beta_j(b_{ij}, g) + b_{ij} \prec_j g + \beta_j(b_{ij}, g)$, then j has an incentive to reject i's offer. In this case, we say that the deviation b_{ij} is *blocked* by $\beta_j(b_{ij}, g)$. We shall *refine the beliefs* that agent j is allowed to hold by imposing a consistency requirement.

Definition 5 *A belief $\beta_j(b_{ij}, g)$ is said to be consistent if $g + \beta_j(b_{ij}, g) + b_{ij} \succ_i g + \beta_j(b_{ij}, g)$.*

Agent j's belief $\beta_j(b_{ij}, g)$ about the other contract signed by i is consistent with the contract b_{ij} that agent j observed if the addition of b_{ij} to $g + \beta_j(b_{ij}, g)$ makes agent i better off.

We view consistency as a mild restriction that any reasonable belief must satisfy.[3] There are several ways to refine the set of admissible beliefs even further. For instance, it is possible to add a requirement that agent j's conjectured contract renders both i and k better off (such a requirement would make it necessary for us to specify k's beliefs). Placing additional constraints on the set of permissible beliefs would enlarge the set of potential deviations. Thus, our conservative approach results in a relatively simple framework and it allows us to interpret our results as the limits to risk-sharing and risk-bearing.

Underlying the notion of consistency are *forward-induction* considerations in the spirit of the Nash equilibrium refinements literature and, in particular, the intuitive criterion (Cho and Kreps, 1987). If we think of the conjectured deviation as the "type" of a deviation's initiator i, then consistency implies that an offer's receiver j must believe that the type of the initiator is one that can benefit from making this offer. Thus, weak stability incorporates considerations from the Nash equilibrium refinements literature into a notion of stability in the spirit of cooperative game theory.

Definition 6 *A contract g is said to be weakly stable if, for each $i \in I$ and contract b_{ij} such that $g+b_{ij} \succ_i g$, there exists a consistent belief $\beta_j(b_{ij}, g)$ such that $g+\beta_j(b_{ij}, g) \succ_j g+\beta_j(b_{ij}, g)+b_{ij}$.*

Observe that a deviation consists of a bilateral contract and the agent who initiates the contract. The same contract is treated differently when the identity of its initiator is different. If $g+b_{ij} \succ_i g$ and $g+b_{ij} \succ_j g$, then the weak stability of g requires that the side-contract b_{ij} be blocked both by a consistent belief $\beta_j(b_{ij}, g)$ and by a consistent belief $\beta_i(b_{ij}, g)$.

Discussion: Solution concept

We shall now discuss a few variants of weak stability and their potential implications for our results. But before we do so, it may be beneficial to discuss the possibility of using a few "off the shelf" notions of stability to solve the model. For example, when considering deviations by a coalition, it is natural to think of cooperative notions such as the core or the Aumann–Maschler bargaining set (Aumann and Maschler, 1964). However, such notions cannot be used to examine the effect of *adding new contracts to an existing set* of contracts as the idea underlying such notions is that coalitions deviate

[3] A consistent belief need not exist. For example, agent j cannot form any belief that is consistent with a side-contract b_{ij} in which i pays j the same positive amount in both realisations of the signal. Such cases are of minor interest since side-contracts that cannot be rationalised do not violate the pairwise stability of the multilateral contract.

to a state of autarchy. This is also the idea underlying the self-enforcing risk-sharing agreements in Genicot and Ray (2003). Concepts such as strong Nash equilibrium (Aumann, 1959), coalition-proof Nash equilibrium (Bernheim, Peleg, and Whinston, 1987), and coalitional rationalisability (Ambrus, 2006) can capture the idea that a coalition of agents deviates while agents who are not members of the deviating coalition do not change their behaviour. However, as these concepts are noncooperative, they require strong assumptions about the way contracts and deviations are negotiated. Moreover, the existence of a strong Nash equilibrium or a coalition-proof Nash equilibrium is not guaranteed.

Cancelling contracts unilaterally

Weak stability does not include the possibility of unilaterally cancelling a signed contract. Allowing agents to cancel a previously signed contract allows for more deviations than the present notion of stability does. However, it does not add new beliefs that can disqualify deviations. To see this, consider an agent j who receives an offer to sign a bilateral contract with an agent i and rejects it based on a conjecture that i cancelled a contract $b_{ik}, k \in I - \{i, j\}$. This belief is equivalent to a belief that i and k wrote a side-contract $\hat{b}_{ik}$ such that $\hat{b}_{ik}(s) = -b_{ik}(s)$ for each $s \in S$. In fact, it is possible to show that unilateral cancellation of contracts does not change any of our results. However, incorporating it into the model would require us to elaborate on the structure of the multilateral contract and this would significantly complicate the exposition.

Beliefs that consist of a profile of side-contracts

Under weak stability, the belief formed by an agent who receives an offer to deviate consists of one additional side-contract. Our focus on IC contracts allows us to check only two deviations (one real and one conjectured) such that we can make relatively general assumptions about the set of admissible contracts and the manipulation function.

Alternatively, we can let an agent who receives an offer to deviate hold a belief that consists of a profile of side-contracts. To do so, we need to extend our assumptions about the manipulation function in order to fully pin down the outcomes in cases in which there are more than two agents whose incentive-compatibility constraints are violated. There are natural extensions of our assumptions that do not change the analysis or the main insights that are gained from the model.

Existence and Main Result

Proposition 3 ***(Existence)*** *Suppose that $|M| \geq 2$. There exists a contract that is weakly stable, IC, and IR.*

The proof shows that the null contract is always weakly stable, which illustrates how mild the consistency requirement is. The set of weakly stable contracts typically includes other contracts as well, as we shall illustrate in the next section. The multiplicity of weakly stable contracts is not surprising as even stronger concepts such as pairwise stability do not guarantee uniqueness in most settings (e.g., matching and network formation). Different weakly stable contracts may induce different levels of risk-sharing and risk-bearing. In the next section, we shall explore the limits to the volume of trade that can be obtained by means of weakly stable contracts.

Proposition 4 is the present article's main result. It emphasises the tension between stability and efficiency in the context of a conventional risk-sharing economy. The result shows that despite the weakness of the solution concept and the fact that the set of weakly stable contracts can be large, under fairly general conditions, there exists no contract that is both weakly stable and constrained-efficient.

Proposition 4 ***(Main result)*** *Suppose that for each $i \in I$ it holds that $\pi_i = \pi \in (0, 1)$ and u_i is strictly concave. Moreover, assume that richness and nontriviality are satisfied. If g is weakly stable, then it is not constrained-efficient.*

In the proof we describe one deviation that cannot be blocked by any consistent belief and show that if the multilateral contract is constrained-efficient, then this deviation exists. The deviation includes a side-contract by which an agent who can manipulate the signal *colludes* with an agent who cannot do so in order to set the signal to their preferred realisation ex post. In the collusive side-contract, the agent who cannot manipulate the signal makes positive signal-contingent payments to the manipulator. These payments incentivise the latter to manipulate the signal ex post, if necessary, since they violate his incentive-compatibility constraint.

The fact that the above deviation involves agents with *heterogeneous strategic capabilities* plays a key role. In particular, it is important that the deviation is initiated by an agent who cannot manipulate the signal. The inability of the deviation's initiator to manipulate the signal ex post affects the beliefs held by the receiver of an offer to sign a side-contract. Intuitively, it is harder for the receiver to suspect that the initiator has an ulterior motive when the initiator is not a manipulator. In the present model, agents are suspicious of offers to deviate made by manipulators and are less inclined to accept such offers.

In order to illustrate this, suppose that agent $i \notin M$ offers agent $m \in M$ the opportunity to sign a side-contract with the intention that m will manipulate the signal ex post. Since i is not a manipulator, agent m will not suspect that if he rejects i's offer, then i will manipulate the signal himself. If, on the contrary, i were a manipulator, then m might reject a similar offer to deviate based on the belief that if he rejects i's offer then i will manipulate the signal himself. For example, m might suspect that i has an ulterior motive in the form of another side-contract that incentivises i to manipulate the signal ex post (i.e., m might believe that i is trying to free-ride on him and make him pay the cost of manipulation ex post instead of i doing so himself).

4 Applications

In this section, we present two applications of the model. In the first application we study risk-sharing in a reinsurance market and in the second application we study risk-bearing when agents are motivated by different prior beliefs. The critical parameter in both applications is the proportion of agents who can manipulate the contractible variable. The main message of both of the applications is that the maximal level of risk-sharing (or risk-bearing) that can be sustained using a weakly stable contract is U-shaped (or V-shaped) w.r.t. the share of manipulators. That is, an increase in the proportion of manipulators does not necessarily imply a reduction in the volume of trade that can be sustained by means of a weakly stable contract.

4.1 Reinsurance

We study a reinsurance market in which local insurers who are exposed to a local shock receive coverage from external reinsurers who are not directly exposed to the shock. The contractible variable is a local regulator's declaration of a state of emergency. Reinsurance contracts and instruments (e.g., catastrophe bonds) are typically contingent on such state-dependent signals and not on actual losses incurred by insurers, in order to prevent moral hazard problems in underwriting and claim settlements (see Doherty, 1997).

We assume that some of the local insurers can manipulate the contractible variable by lobbying the regulator and that the external insurers cannot do so. We interpret the fraction of local insurers who have the ability to influence the regulator's decision as a proxy for *the level of corruption* in the economy. That is, the larger the fraction of insurers who can access the regulator, the more widespread the corruption. We examine how the *maximal level of coverage* that can be provided to the local insurers using an IR, IC, and weakly stable contract is affected by the primitives of the model.

We partition the set of agents I into a set of local insurers L and a set of external reinsurers E and assume that $M \subseteq L$ (the results in this section are robust to letting the external insurers manipulate the signal by incurring some cost as long as there are sufficiently many such reinsurers). To capture the idea that the local insurers are exposed to the same high-volume shock, we set $w_i(H) - w_i(L) = w \geq c$ for each $i \in L$. We shall assume that the cardinality of E is large relative to that of L such that the external reinsurers can absorb all the risk in the economy. Specifically, we assume that $\frac{w}{c} < \frac{|E|}{|L|}$. This condition implies that even if the members of E were to provide full coverage to the members of L (and spread this coverage equally among them), each $i \in E$ would hold a position $g_i(h) - g_i(l) \leq c$. Observe that richness and nontriviality do not hold since the agents face an aggregate shock.

To avoid frictions arising from the discreteness of L, we assume that there are many local insurers and denote the share of manipulators $\frac{|M|}{|L|}$ by α. For the sake of tractability, we assume that the local insurers exhibit constant absolute risk aversion (CARA). That is, for each $i \in L$, $u_i(z) = -exp(-\gamma z)$, $\gamma > 0$. To simplify the exposition, it is also assumed that each $i \in E$ is risk neutral.[4] Observe that under CARA, an agent's marginal rate of substitution between wealth in both states is pinned down by his exposure $w_i(H) - w_i(L) + g_i(h) - g_i(l)$; for instance, it is $exp[-\gamma w]$ in the case where $i \in L$ is not covered (i.e., $g_i(h) = g_i(l) = 0$). To simplify[5] the exposition, we strengthen Assumption 2 by assuming that $prob\{s = h|\theta = H, g\} = prob\{s = h|\theta = L, g\} = 0.5$ for each $g \in G$ such that $|PM(g)| = 2$ and $g_m(h) - g_m(l) > 0 > g_{m'}(h) - g_{m'}(l)$ for a pair of agents $m, m' \in PM(g)$.

We start by showing that even though richness does not hold, if $\alpha \in (0,1)$, then weakly stable constrained-efficient contracts do not exist. Then, we show that the maximal level of risk that can be shared by means of a weakly stable contract is U-shaped in α. In Appendix B, we provide a closed-form solution to this maximal level, and show that it can be significantly lower than the constrained-efficient level of coverage.

The next result is based on an argument similar to the one used in Proposition 4 and does not rely on the CARA assumption.

Proposition 5 *If $\alpha \in (0,1)$, then there exists no contract that is both constrained-efficient and weakly stable.*

We now examine the effect of the level of corruption in the economy on the *cov-*

[4] As long as the cardinality of E is sufficiently large, all of the analysis goes through under the assumption that the members of E have the same preferences as the members of L.

[5] All of the results presented in this section hold when this assumption is relaxed; a full proof is available upon request.

erage that the local insurers can obtain. We define the total coverage as $\sum_{i \in L} min$ $\{g_i(l) - g_i(h), w\}$, where insurer i's ($i \in L$) coverage is $min\{g_i(l) - g_i(h), w\}$. We now study the maximal total coverage that the local insurers can attain using an IR, IC, and weakly stable contract as a function of the share of manipulators α.

Proposition 6 *Suppose that* $|M| \geq 2$. *There exists an* $\alpha^* \in (0,1)$ *such that the maximal total coverage that can be obtained using an IR, IC, and weakly stable contract is increasing (respectively, decreasing) in* α *for* $\alpha > \alpha^*$ *(respectively,* $\alpha < \alpha^*$*).*

Proposition 6 establishes that the maximal total coverage that can be obtained by means of a weakly stable contract is U-shaped in the share of manipulators. A possible interpretation of this result is that increasing the level of corruption increases the maximal level of coverage that can be provided to the local insurers when corruption is widespread in the economy.

The proof shows that a multilateral contract is weakly stable if and only if for each pair of agents, $i \in L - M$ and $m \in M$, the manipulation cost c is not exceeded by the sum of the coverage provided to m and i's willingness to pay to guarantee that the signal will be l. These constraints are given in (7) in the proof. At the optimum, each of the manipulators obtains the same coverage $g_m(l) - g_m(h)$ and each of the nonmanipulators obtains the same coverage $g_i(l) - g_i(h)$, which allows us to reduce these constraints to one constraint, which is given in (9).

Essentially, we maximise a convex combination (with weights α and $1 - \alpha$) of the coverage provided to manipulators and the coverage provided to nonmanipulators subject to (9), which is concave. Thus, the minimum of the problem with respect to α is obtained at $\alpha^\star \in (0,1)$ for which the coverage provided to each manipulator equals the coverage provided to each local agent who cannot manipulate the contractible variable (i.e., $g_m(l) - g_m(h) = g_i(l) - g_i(h)$). If $\alpha > \alpha^\star$, then the manipulators' coverage $g_m(l) - g_m(h)$ is greater than the nonmanipulators' coverage $g_i(l) - g_i(h)$ and the total coverage is increasing in the share of manipulators α. Analogously, if $\alpha < \alpha^\star$, then the manipulators' coverage $g_m(l) - g_m(h)$ is less than the nonmanipulators' coverage $g_i(l) - g_i(h)$ and the total coverage is decreasing in the share of manipulators α.

Comparative statics: Risk aversion

Let us examine the effect of risk aversion on the level of risk-sharing that can be sustained by means of a weakly stable contract. Fix a contract g and consider the maximal willingness of an agent $i \in L - M$ to pay to guarantee that the signal will be l, which is given on the RHS of (9). It is lower for greater values of the coefficient of risk aversion γ. Intuitively, when agent $i \in L - M$ incentivises agent $m \in M$ to manipulate the signal by paying him x if $s = l$, it is as if agent i were giving up on state-dependent coverage of $g_i(l)$ in return for a sure transfer of $g_i(l) - x$. When agent i becomes more averse to risk, the state-dependent coverage becomes more attractive than the sure transfer, such that i's willingness to pay for manipulation decreases. Increasing the risk-aversion parameter relaxes (9) and allows more coverage to be provided.

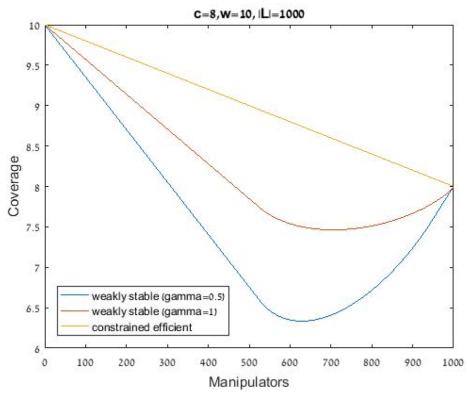

Figure 1: $c = 8$

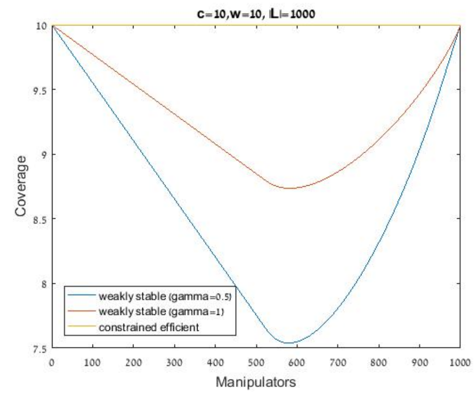

Figure 2: $c = 10$

In Appendix B we illustrate the effects of α and γ on the level of coverage by solving the model analytically. The primary goal of the analysis that is provided in the Appendix is to show that the adverse effect on the level of coverage is of a first-order magnitude. In Figure 1 we illustrate the maximal average level of coverage for $w = 10$, $c = 8$, $|L| = 1000$, $\gamma = 0.5$, and $\gamma = 1$ versus the constrained-efficient level of coverage. Figure 2 illustrates the results for $w = 10$ and $c = 10$, and demonstrates that even when $c = w$ such that in a constrained-efficient contract the agents are fully covered, the maximal level of coverage that can be obtained using a weakly stable contract is significantly lower than the constrained-efficient level of coverage for some values of α. Note that since side-contracts signed by two manipulators cannot destabilise a multilateral contract, at the extremes (i.e., for $\alpha \in \{0, 1\}$), the maximal level of coverage that can be obtained using an IR, IC, and weakly stable contract coincides with the constrained-efficient level of coverage.

4.2 Speculative trade

In this subsection, we study reallocation of risk that is motivated by different prior beliefs about the state of nature. To focus on speculation, we assume that all of the agents in our economy are risk neutral.

Remark 1 *If there are two agents $i, j \notin M$ such that $\pi_i \neq \pi_j$, then there exists no constrained-efficient contract.*

Remark 1 follows from the well-known fact that risk-neutral agents who hold different prior beliefs are always better off increasing the stakes of a bet between them (e.g., by writing an additional side-contract).

We define the *volume of speculative trade* in a contract g to be $\sum_{i \in I} |g_i(h) - g_i(l)|$. The next result establishes that the volume of speculative trade that can be sustained using an *IC and weakly stable contract* is bounded from above.

Proposition 7 *Let $|M| \geq 2$. The volume of speculative trade that can be sustained by means of a contract that is both IC and weakly stable is bounded from above.*

The proof relies on the combination of incentive compatibility and weak stability. First, incentive compatibility bounds the speculative positions of the members of M (i.e., for each $m \in M$, $|g_m(h) - g_m(l)| \leq c$). Weak stability implies that there is no nonmanipulator who is willing to pay $c - (max_{m \in M} g_m(h) - g_m(l))$ to guarantee that the signal will be h and there is no nonmanipulator who is willing to pay $c - (max_{m \in M} g_m(l) - g_m(h))$ to guarantee that the signal will be l. This bounds each agent i's ($i \notin M$) speculative position from above by $max\left\{\frac{2c}{\pi_i}, \frac{2c}{1-\pi_i}\right\}$.

Let us impose more structure in order to examine the maximal volume of speculative trade that can be sustained by means of weakly stable contracts. We assume that there are two types of agents. These types differ from each other in their prior beliefs about the state of nature, whereas the share of manipulators in each type of agent is identical. Formally, we partition I into two disjoint groups of equal size, I^h and I^l, and assume that each $i \in I^l$ has a prior belief π_l and each $i \in I^h$ has a prior belief $\pi_h > \pi_l$. We assume that $|M \cap I^h| = |M \cap I^l|$. As in the previous subsection, we strengthen Assumption 2 by assuming that $prob\{s = h|\theta = H, g\} = prob\{s = h|\theta = L, g\} = 0.5$ for each $g \in G$ such that both $|PM(g)| = 2$ and $g_m(h) - g_m(l) > 0 > g_{m'}(h) - g_{m'}(l)$ for a pair of agents $m, m' \in PM(g)$. To avoid integer problems, let us define $\alpha := \frac{|M|}{|I|}$.

We now derive a closed-form solution to the maximal volume of speculative trade that can be obtained using IR, IC, and weakly stable contracts. To examine speculation among agents with non-common priors, we assume that the following condition is satisfied.

Condition 1 *The multilateral contract g is said to satisfy Condition 1 if $g_i(h) \geq g_i(l)$ for each $i \in I^h$ and $g_i(l) \geq g_i(h)$ for each $i \in I^l$.*

Proposition 8 *Let $\alpha \in (0,1)$. The maximal volume of speculative trade that can be sustained by means of an IR, IC, and weakly stable contract that satisfies Condition 1 is*

$$n * max\left\{\alpha c, min\left\{\frac{c(1-\alpha)}{1-\pi_h}, \frac{c(1-\alpha)}{\pi_l}\right\}\right\}.$$

The maximal volume of speculative trade is V-shaped in the proportion of manipulators. Even though the agents' preferences are linear, the maximal volume of speculative trade is either *strictly increasing* in α or *strictly decreasing* in α. The maximal volume of speculative trade is weakly increasing in π_h and weakly decreasing in π_l. That is, for a given cost of manipulation c, when the agents' beliefs are more polarised, there is room for more speculative trade.

The intuition behind the V-shaped volume of trade is similar to the intuition behind the U-shaped level of coverage obtained in Proposition 6. There are two differences, however. First, the linearity of the utility functions implies that the willingness to pay to impose one's preferred signal is linear in one's speculative position, which implies a V-shaped upper bound rather than a U-shaped one. The second key difference is that there are no external agents who can absorb the positions of the members of I^h or I^l, which results in the minimum in the expression for the volume of trade.

The effect of polarisation: A comparison to bilateral trade

As a benchmark, consider the case of $n = 2$ with risk-neutral agents. The magnitude of the difference between the agents' prior beliefs has no effect on the volume of trade. If $\pi_i \neq \pi_j$ and one of the agents can manipulate the signal, then the volume of trade is $2c$. If $\pi_i \neq \pi_j$ and none of the agents can manipulate the signal, then the volume of trade is not bounded (i.e., i and j will always want to scale up the stakes of the bets between them such that there is no upper bound on the volume of trade). In conclusion, the values of π_i and π_j do not affect the volume of trade that can be sustained.

When $n > 2$, there are contractual externalities and the agents' ex-ante willingness to pay to guarantee their preferred realisation of the signal ex post plays a key role. In particular, we can observe in (10) that the sum of the maximal willingness to pay for manipulation of an agent $i \notin M$ and the maximal speculative position held by a member of M cannot exceed c.

Agent i's willingness to pay to guarantee that ex post the signal will be l is $\pi_i (g_i(l) - g_i(h))$. It is increasing in π_i since i benefits from a collusive side-contract that guarantees his preferred realisation only when there is ex-post manipulation, that is, only when the state is H. The less i believes that state H is likely, the less i is willingness to pay to impose realisation l ex post and, therefore, the greater the speculative position $g_i(l) - g_i(h)$ that i can hold. Analogously, the less i believes that state L is likely, the greater the speculative position $g_i(h) - g_i(l)$ that i can hold without violating the weak stability of the contract. Thus, the agents' beliefs affect the maximal volume of speculative trade that can be obtained by means of a weakly stable contract.

5 Extensions and Modifications

A private shock

Our assumption about the richness of the economy ruled out cases in which one agent is exposed to a shock and he is the only one who can manipulate the signal. For instance, consider insurance contracts, which are typically conditioned on an insuree's report about the occurrence of the shock. The cost of reporting that a shock occurred when it did not occur is c. The interpretation of the assumption that $n > 2$ is that the agent can purchase coverage from multiple insurers.

Since the domain is a simple variant of the one presented in Section 4.1, we omit its formal presentation and instead state the following claim.

Claim 1 *Let $M = \{i\}$ and $w_j(H) = w_j(L)$ for every $j \in I - \{i\}$. Every constrained-efficient contract is pairwise stable.*

In a constrained-efficient contract, the agent who is exposed to the shock purchases coverage from the insurers in return for a premium. Such a multilateral contract must be pairwise stable since a collusive side-contract that incentivises the agent to manipulate the contractible variable by submitting a false report cannot make any insurer better off.

Asymmetric manipulation costs

Let us relax the assumption that the cost of manipulation is identical for different agents and different signals. Suppose instead that each $i \in M$ can change the signal's realisation from s to s' by paying a cost of $c_i(s \to s')$. What is the effect of this modification on the results obtained in the paper? Propositions 1–4, 5, and 7 do not directly depend on the symmetry assumptions. Under some adjustments all of these results hold. For instance, the definition of nontriviality should be adjusted to the following: an economy is said to satisfy nontriviality if there exist two agents $i, j \in M$ such that $w_i(H) - w_i(L) \geq c_i(h \to l)$ and $w_j(L) - w_j(H) \geq c_j(l \to h)$.

More than two states and signals

Let us assume that $\Theta = \{1, ..., t\}$ and $S = \{1^\star, ..., t^\star\}$ and that unless the signal is manipulated, it is $k^\star$ if and only if the state is k. Denote the cost of manipulating the signal from s to s' by $c(s \to s')$ and set $c(s \to s) := 0$. Intuitively, in an IC contract, it must be that $g_i(s') - g_i(s) \leq c(s \to s')$ for each $i \in M$ and $s, s' \in S$.

We need to adapt our assumptions about manipulation (that is, the assumptions about how the signal is set and who incurs the cost of manipulation) to include more than two states and two signals. One possible modification is that if $PM(g) = \emptyset$, then there is no manipulation. Otherwise, nature chooses a member of M who gets an opportunity to set the signal to his preferred realisation and to incur the cost of manipulation, if there is any.

By adapting the nontriviality and richness assumptions to include more than two states and signals, our negative results remain essentially the same. It is also possible to show the existence of weakly stable multilateral contracts. Propositions 6 and 8 are more subtle and require more details.

6 Concluding Remarks

The main contribution of the paper is fourfold. First, we incorporate the idea of manipulation into multilateral risk-sharing and speculative trade. Second, our substantive results establish that when it is possible to manipulate the contractible variable, reallocation of risk is highly constrained by the agents' ability to write side-contracts. Third, we contribute to the network formation literature by analyzing a network of contracts with a new externality that results from the ability to manipulate the contractible variable. Finally, at the methodological level, we introduce a coarsening of pairwise stability in the tradition of cooperative game theory that incorporates insights from the Nash equilibrium refinements literature.

Comment: Analogy to network formation

Throughout the paper we studied the properties of one multilateral contract while ignoring its structure. The assumption that the deviations consist of bilateral contracts suggests an intuitive structure of the multilateral contract: a network of IR bilateral contracts. The natural question to ask is whether the restriction to bilateral contracting constrains reallocation of risk. A result obtained by Rader (1968) shows that this restriction is not a constraint on the agents' ability to share risk: there always exists a constrained-efficient multilateral contract that can be decomposed into a collection of IR bilateral contracts.

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Appendix A: Proofs

The proofs of Propositions 1 and 2 are based on the following lemma.

Lemma 1 *Let g be an IC multilateral contract. If there exists a pair of agents, $m \in M$ and $i \neq m$, such that $|g_m(h) - g_m(l)| = c$ and $sign(g_m(h) - g_m(l)) = sign(g_i(h) - g_i(l))$, then g is not pairwise stable.*

Proof. Without loss of generality, assume that $g_m(h) - g_m(l) = c$ and $g_i(h) > g_i(l)$. Consider a side-contract b_{mi} such that $c \geq b_{mi}(h) > b_{mi}(l) = 0$. Observe that $g_m(h) - g_m(l) + b_{mi}(h) > c$ and $c > g_i(h) - g_i(l) - b_{mi}(h) > -c$. Since g is IC, it follows that $|g_z(h) - g_z(l)| \leq c$ for every $z \in I - \{i, m\}$. Thus, $PM(g + b_{mi}) = \{m\}$. By Assumption 1, the multilateral contract $g + b_{mi}$ results in $s = h$ regardless of the state of nature. If $b_{mi}(h)$ is sufficiently close to 0, then $g + b_{mi} \succ_i g$ as $\pi_i g_i(h) +$

$(1-\pi_i)\, g_i(l) < g_i(h) - b_{mi}(h)$. Agent m is better off signing the side-contract b_{mi} since $\pi_m g_m(h) + (1-\pi_m)\, g_m(l) < \pi_m (g_m(h) + b_{mi}(h)) + (1-\pi_m)(g_m(h) - c + b_{mi}(h))$. Since both counterparties are better off signing b_{mi}, g is not pairwise stable. ■

Proof of Proposition 1

Assume by way of contradiction that g is IC and pairwise stable. First, we focus on the case where $|I - M| \leq 1$. Consider three agents $k, m, i \in M$ and without loss of generality assume that $\pi_k > \pi_i > \pi_m$. We now show that it must be that $g_k(h) - g_k(l) = c$ and $g_m(l) - g_m(h) = c$.

Suppose that $g_m(l) - g_m(h) < c$ and $g_i(h) - g_i(l) < c$. We now construct a side-contract b_{mi} such that $g + b_{mi} \succ_i g$ and $g + b_{mi} \succ_m g$. Let $b_{mi}(l) = \epsilon$ and $b_{mi}(h) = \frac{\hat{\pi}-1}{\hat{\pi}}\epsilon$, where $\hat{\pi} \in (\pi_m, \pi_i)$. If $\epsilon > 0$ is sufficiently small, then $g + b_{mi}$ is IC. Both agents are better off adding the side-contract to g since $-(1-\pi_i)\,\epsilon - \pi_i \frac{\hat{\pi}-1}{\hat{\pi}}\epsilon > 0$ and $(1-\pi_m)\,\epsilon + \pi_m \frac{\hat{\pi}-1}{\hat{\pi}}\epsilon > 0$. This contradicts the pairwise stability of g and, therefore, $g_m(l) - g_m(h) = c$ or $g_i(h) - g_i(l) = c$. By the same argument, $g_i(l) - g_i(h) = c$ or $g_k(h) - g_k(l) = c$. If $g_k(h) - g_k(l) = c$ and $g_i(h) - g_i(l) = c$, then, by Lemma 1, g is not pairwise stable. Also, if $g_i(h) - g_i(l) = -c$ and $g_m(h) - g_m(l) = -c$, then, by Lemma 1, g is not pairwise stable. Since g is pairwise stable and IC, it follows that $g_k(h) - g_k(l) = c$ and $g_m(l) - g_m(h) = c$.

Since $n > 3$, there is an agent $i' \in I - \{i, m, k\}$ such that $\pi_i \neq \pi_{i'}$. If $g_i(h) - g_i(l) = 0 = g_{i'}(h) - g_{i'}(l)$; then we can construct a side-contract $b_{ii'}$ (similar to b_{mi} described above) that makes both i and i' better off. However, since g is pairwise stable, it is impossible to construct such a side-contract and, therefore, $|g_i(h) - g_i(l)| > 0$ or $|g_{i'}(h) - g_{i'}(l)| > 0$. Recall that $g_k(h) - g_k(l) = c$ and $g_m(l) - g_m(h) = c$. By Lemma 1, it follows that g is not pairwise stable.

To complete the proof, consider the case where $|I - M| > 1$. Observe that if there exist two agents $i, m \notin M$ such that $\pi_i > \pi_m$, then there exists a contract b_{mi}, similar to the contract b_{mi} described above, such that $g + b_{mi}$ is IC, $g + b_{mi} \succ_i g$, and $g + b_{mi} \succ_m g$. Hence, g is not pairwise stable.

Proof of Proposition 2

Suppose that g is IC and pairwise stable. By nontriviality, there exist two agents $m, m' \in M$ such that $w_m(H) - w_m(L) \geq c$ and $w_{m'}(L) - w_{m'}(H) \geq c$. If $g_m(l) - g_m(h) < c$ and $g_{m'}(h) - g_{m'}(l) < c$, then $w_m(H) + g_m(h) - w_m(L) - g_m(l) > 0 > w_{m'}(H) + g_{m'}(h) - w_{m'}(L) - g_{m'}(l)$. In this case, m and m' would be better off signing a side-contract $b_{mm'}$ in which they provide each other with fair insurance (the stakes of the contract $b_{mm'}$ can be set to be small such that $g + b_{mm'}$ is IC and there is no manipulation). It follows that if g is IC and pairwise stable, then $g_m(l) - g_m(h) = c$ or $g_{m'}(h) - g_{m'}(l) = c$.

Without loss of generality, assume that $g_m(l) - g_m(h) = c$. By Lemma 1, if there exists an agent $j \neq m$ such that $g_j(l) > g_j(h)$, then g is not pairwise stable. Thus, $g_i(l) \leq g_i(h)$ for each $i \in I - \{m\}$. By richness, there exists an agent $i \notin M$ such that $w_i(H) - w_i(L) > 0$. Since $g_i(l) \leq g_i(h)$, it follows that $w_i(H) + g_i(h) - w_i(L) - g_i(l) > 0$. If there exists an agent $j \notin M$ such that $w_j(H) + g_j(h) - w_j(L) - g_j(l) < 0$, then i and j are better off writing a side-contract b_{ij} in which they provide each other with fair insurance (note that $g + b_{ij}$ is IC as $i, j \notin M$). By richness, there exists an agent $j \notin M$ such that $w_j(L) > w_j(H)$. By the previous argument, pairwise stability of g requires that $w_j(H) + g_j(h) - w_j(L) - g_j(l) \geq 0$. Thus, $g_j(h) > g_j(l)$.

By nontriviality, there exists an agent $m' \in M - \{m\}$ such that $w_{m'}(L) - w_{m'}(H) \geq c$. Since there exists an agent $i \notin M$ such that $w_i(H) + g_i(h) - w_i(L) - g_i(l) > 0$, it follows that if g is pairwise stable, then $g_{m'}(h) - g_{m'}(l) = c$ (otherwise m' and i would be better off writing a side-contract in which they provide each other with fair insurance without violating m''s incentive-compatibility constraint).

In conclusion, there exist an agent $m \in M$ such that $g_m(l) - g_m(h) = c$, an agent $m' \in M$ such that $g_{m'}(h) - g_{m'}(l) = c$, and an agent $j \notin M$ such that $g_j(h) > g_j(l)$. By Lemma 1, this is in contradiction to the pairwise stability of g.

Proof of Proposition 3

Let g' be a multilateral contract such that $g'_i(h) = g'_i(l) = 0$ for each $i \in I$. We shall show that g' is weakly stable. Consider a side-contract b_{ij} such that $g' + b_{ij} \succ_i g'$. Our objective is to find a consistent belief $\beta_j(b_{ij}, g')$ that blocks b_{ij}. There are two cases to examine: (i) $g' \succ_j g' + b_{ij}$ and (ii) $g' \not\succ_j g' + b_{ij}$. Consider case (i) and observe that $\beta_j(b_{ij}, g') = \emptyset$ is consistent since $g' + b_{ij} \succ_i g'$. Since $g' \succ_j g' + b_{ij}$, it follows that $\beta_j(b_{ij}, g') = \emptyset$ blocks the deviation b_{ij}.

Consider case (ii) and assume that $g' + b_{ij}$ is not IC. Since $g'_i(h) = g'_i(l) = 0$ for each $i \in I$, it follows that $PM(g' + b_{ij}) \subseteq \{i, j\}$. By Assumptions 1 and 2, if $|PM(g' + b_{ij})| \in \{1, 2\}$, then the realisation of the signal ex post is independent of the realised state. Since b_{ij} induces zero-sum transfers between i and j independently of the state and both agents are risk averse, we obtain a contradiction to the assumption that $g' + b_{ij} \succ_i g'$ and $g' \not\succ_j g' + b_{ij}$.

Consider case (ii) and assume that $g' + b_{ij}$ is IC. Since $g' + b_{ij} \succ_i g'$ and $g' + b_{ij} \succ_j g'$, it follows that $b_{ij}(s) > 0 > b_{ij}(s')$, where $s' \neq s$. Without loss of generality, assume that $b_{ij}(h) > 0 > b_{ij}(l)$, and split the analysis into two separate cases: either $i \in M$ or $i \notin M$. Suppose that $i \in M$ and consider the belief $\beta_j(b_{ij}, g') = b_{ik}$ such that $b_{ik}(h) - b_{ik}(l) = c$. Since $g' + b_{ij}$ is IC, it follows that $PM(g' + b_{ik} + b_{ij}) = \{i\}$. Moreover, $PM(g' + b_{ik}) = \emptyset$. By Assumption 1, in both states, the signal resulting from $g' + b_{ik} + b_{ij}$ is h. Thus,

$$\pi(w_i(H) + b_{ij}(h) + b_{ik}(h)) + (1-\pi)(w_i(L) + b_{ij}(h) + b_{ik}(h) - c) > \\ \pi(w_i(H) + b_{ik}(h)) + (1-\pi)(w_i(L) + b_{ik}(l))$$

and

$$\pi(w_j(H) - b_{ij}(h)) + (1-\pi)(w_j(L) - b_{ij}(h)) < \\ \pi w_j(H) + (1-\pi) w_i(L).$$

The first inequality implies that $\beta_j(b_{ij}, g')$ is consistent (recall that $g'_i(h) = g'_i(l) = 0$) and the second inequality implies that $g' + b_{ik} \succ_j g' + b_{ik} + b_{ij}$.

To complete the analysis, suppose that $i \notin M$ and consider a belief $\beta_j(b_{ij}, g') = b_{ik}$ such that $k \in M - \{j\}$ and $b_{ik}(l) - b_{ik}(h) > c$. Since $g' + b_{ij}$ and g' are IC, $PM(g' + b_{ik} + b_{ij}) = PM(g' + b_{ik}) = \{k\}$. By Assumption 1, in both states, the signal resulting from $g' + b_{ik} + b_{ij}$ is h. Moreover, by Assumption 1, in both states, the signal resulting from $g' + b_{ik}$ is h. Thus, given b_{ik}, j (respectively, i) is only affected by the downside (respectively, upside) of b_{ij}. It follows that $\beta_j(b_{ij}, g')$ is consistent with b_{ij} and $g' + b_{ik} \succ_j g' + b_{ik} + b_{ij}$.

Proof of Proposition 4

The first step of the proof shows that if richness and nontriviality are satisfied and g is constrained-efficient, then there exist an agent $m \in M$ such that $|g_m(h) - g_m(l)| = c$ and an agent $i \notin M$ such that $sign(g_m(h) - g_m(l)) = sign(g_i(h) - g_i(l))$.

Step 1. Nontriviality implies that there exist two agents $m, m' \in M$ such that $w_m(H) - w_m(L) \geq c > -c \geq w_{m'}(H) - w_{m'}(L)$. If $g_m(l) - g_m(h) < c$ and $g_{m'}(h) - g_{m'}(l) < c$, then m and m' can write a side-contract $b_{mm'}$ in which they provide each other with fair insurance without violating any IC constraint (i.e., $g + b_{mm'}$ is IC). Both m and m' are better off signing this side-contract. Observe that the constrained efficiency of g implies that there exists no bilateral contract $b_{mm'}$ such that $g + b_{mm'}$ is IC, $g + b_{mm'} \succ_{m'} g$, and $g + b_{mm'} \succ_m g$. It follows that if g is constrained-efficient, then $g_m(l) - g_m(h) = c$ or $g_{m'}(h) - g_{m'}(l) = c$.

Without loss of generality, assume that $g_m(l) - g_m(h) = c$. If there exists an agent $i \notin M$ such that $g_i(l) > g_i(h)$, then we have found a pair of agents, $m \in M$ and $i \notin M$, such that $|g_m(h) - g_m(l)| = c$ and $sign(g_m(h) - g_m(l)) = sign(g_i(h) - g_i(l))$. Suppose that for each $i \notin M$, $g_i(h) \geq g_i(l)$.

By richness, there exists an agent $i \notin M$ such that $w_i(H) > w_i(L)$. Since $g_i(h) \geq g_i(l)$, it follows that $w_i(H) + g_i(h) - w_i(L) - g_i(l) > 0$. If there exists an agent $j \notin M$ such that $w_j(H) + g_j(h) - w_j(L) - g_j(l) < 0$, then i and j are better off writing a side-contract to provide each other with fair insurance. The side-contract between i and j does not violate any IC constraint and, therefore, it violates the constrained efficiency of g (as $g + b_{ij}$ is IC and Pareto dominates g). Hence, if g is constrained-efficient, then $w_j(H) + g_j(h) - w_j(L) - g_j(l) \geq 0$ for every agent $j \in I - M - \{i\}$. By richness, there exists an agent $j \in I - M - \{i\}$ such that $w_j(L) > w_j(H)$. Thus, $g_j(h) > g_j(l)$.

By nontriviality, there exists an agent $m' \in M - \{m\}$ such that $w_{m'}(L) - w_{m'}(H) \geq c$. Recall that there exists an agent $i \notin M$ such that $w_i(H) + g_i(h) - w_i(L) - g_i(l) > 0$. If g is constrained-efficient, then there exists no side-contract $b_{im'}$ such that: (i) $g + b_{im'}$ is IC, (ii) $g + b_{im'} \succ_i g$, and (iii) $g + b_{im'} \succ_{m'} g$. If $g_{m'}(h) - g_{m'}(l) < c$, there is always a side-contract $b_{im'}$ in which both parties provide each other with fair insurance that satisfies (i), (ii), and (iii). Thus, the constrained efficiency of g implies that $g_{m'}(h) - g_{m'}(l) = c$. Since we already found an agent $j \notin M$ such that $g_j(h) > g_j(l)$, the first part of the proof is completed.

Part 2. Suppose that g is constrained-efficient and, without loss of generality, assume that there exist an agent $m \in M$ such that $g_m(h) - g_m(l) = c$ and an agent $i \notin M$ such that $g_i(h) > g_i(l)$. Consider a side-contract b_{mi} such that $b_{mi}(h) > b_{mi}(l) > 0$. Clearly, $g + b_{mi} \succ_m g$. Since g is constrained-efficient and $g_m(h) - g_m(l) + b_{mi}(h) - b_{mi}(l) > c$, it follows that $PM(g + b_{mi}) = \{m\}$. By Assumption 1, $g + b_{mi}$ results in the signal h in both states. If $b_{mi}(h)$ is sufficiently close to 0, then

$$\pi(w_i(H) + g_i(h)) + (1 - \pi)(w_i(L) + g_i(l)) <$$
$$\pi(w_i(H) + g_i(h) - b_{mi}(h)) + (1 - \pi)(w_i(L) + g_i(h) - b_{mi}(h))$$

and, therefore, $g + b_{mi} \succ_i g$. It is left to show that there exists no consistent belief $\beta_m(b_{mi}, g)$ that blocks b_{mi}.

First, if $g + \beta_m(b_{mi}, g)$ is IC, then it cannot block b_{mi} since m receives a positive transfer from i in both states. Second, suppose that $g + \beta_m(b_{mi}, g)$ is not IC. In this case, $|PM(g + \beta_m(b_{mi}, g))| = 1$. By Assumption 1, the signal that is induced by $g + \beta_m(b_{mi}, g)$ is either h in both states or l in both states. If it is l in both states, then m is better off signing b_{mi} since, by doing so, he guarantees a transfer strictly greater than $g_m(l)$, which is induced by $g + \beta_m(b_{mi}, g)$. If the signal that results from $g + \beta_m(b_{mi}, g)$ in both states is h, then by Assumption 1, the signal that results from $g + \beta_m(b_{mi}, g) + b_{mi}$ is h in both states. This stands in contradiction to the consistency of $\beta_m(b_{mi}, g)$ since the addition of b_{mi} has no effect on the signal in either state (and it induces a positive transfer from i to m in both states).

Proof of Proposition 5

Since the local insurers' preferences satisfy CARA and the reinsurers are risk neutral, it follows that if g is constrained-efficient, then it must be that $w_i(H) - w_i(L) + g_i(h) - g_i(l) = 0$ for each $i \in L - M$. Otherwise, there exists another IC contract g' that is identical to g except that some additional coverage is provided to an insurer $i \in L - M$ by a reinsurer $j \in E$, such that $g' \succ_i g$ and $g' \succ_j g$. By a similar argument, it must be that if g is constrained-efficient, then $g_m(l) - g_m(h) = c$ for each $m \in M$.

Suppose that g is constrained-efficient and consider a side-contract b_{mi} such that $i \in L - M, m \in M, b_{mi}(l) = 2\epsilon > b_{mi}(h) = \epsilon$. Observe that $PM(g + b_{mi}) = \{m\}$. By Assumption 1, the signal that results from $g + b_{mi}$ is l (in both states). If ϵ is sufficiently close to 0, then $g + b_{mi} \succ_i g$ and $g + b_{mi} \succ_m g$. That is, both i and m benefit from this collusion.

It is left to show that there is no consistent belief $\beta_m(b_{mi}, g)$ that blocks b_{mi}. First, if $g + \beta_m(b_{mi}, g)$ is IC, then it cannot block b_{mi} since m receives a positive transfer from i in both states. Second, suppose that $g + \beta_m(b_{mi}, g)$ is not IC. Since $i \notin M$, it must be that $|PM(g + \beta_m(b_{mi}, g))| = 1$. By Assumption 1, the signal that is induced by $g + \beta_m(b_{mi}, g)$ is either h in both states or l in both states. If it is h in both states, then m is better off signing b_{mi} since, by doing so, he guarantees a transfer strictly greater than $g_m(h)$, which is induced by $g + \beta_m(b_{mi}, g)$. If the signal that results from $g + \beta_m(b_{mi}, g)$ in both states is l, then by Assumption 1, the signal that results from $g + \beta_m(b_{mi}, g) + b_{mi}$ is l in both states. This stands in contradiction to the consistency of $\beta_m(b_{mi}, g)$ since the addition of b_{mi} has no effect on the signal in either state (and i pays 2ϵ to m in both states).

Proof of Proposition 6

The first step of the proof (Lemmata 2–5) is to show which deviations can destabilise a multilateral contract, and which deviations cannot do so. We shall now go over all of the possible deviations and show that a necessary and sufficient condition for the weak stability of an IR and IC multilateral contract g is that there exists no side-contract b_{mi} such that $m \in M$, $i \in I - M$, $b_{mi}(h), b_{mi}(l) > 0$, and $g + b_{mi} \succ_i g$.

Lemma 2 shows that we can ignore deviations that include two members of $I - M$.

Lemma 2 *Let g be IC and IR and let $i, j \in I - M$. For each side-contract b_{ij} such that $g + b_{ij} \succ_i g$, there exists a consistent belief $\beta_j(b_{ij}, g)$ such that $g + \beta_j(b_{ij}, g) \succ_j g + \beta_j(b_{ij}, g) + b_{ij}$.*

Proof. Since g is IC and $i, j \in I - M$, it follows that $g + b_{ij}$ is IC. Thus, $g + b_{ij} \succ_i g$ implies that $b_{ij}(h) > 0$ or $b_{ij}(l) > 0$. Without loss of generality, suppose that $b_{ij}(h) > 0$. Consider a belief $\beta_j(b_{ij}, g) = b_{ki}$ such that $k \in M$ and $g_k(h) - g_k(l) + b_{ki}(h) - b_{ki}(l) > c$. Since g is IC and $i, j \notin M$, it must be that $PM(g + b_{ki} + b_{ij}) = PM(g + b_{ki}) = \{k\}$. By Assumption 1, both contracts, $g + b_{ki} + b_{ij}$ and $g + b_{ki}$, result in the signal h in both states of nature. It follows that $g + b_{ki} + b_{ij} \succ_i g + b_{ki}$ and $g + b_{ki} \succ_j g + b_{ki} + b_{ij}$ such that $\beta_j(b_{ij}, g) = b_{ki}$ is consistent with b_{ij} and blocks it. ■

Lemma 3 shows that we can ignore deviations that include two members of M.

Lemma 3 *Let g be IC and IR, and let $m, m' \in M$. For each side-contract $b_{mm'}$ such that $g + b_{mm'} \succ_{m'} g$, there exists a consistent belief $\beta_m(b_{mm'}, g)$ such that $g + \beta_m(b_{mm'}, g) \succ_m g + \beta_m(b_{mm'}, g) + b_{mm'}$.*

Proof. There are four possible cases:

1. $|g_m(h) - g_m(l) + b_{mm'}(h) - b_{mm'}(l)| > c \geq |g_{m'}(h) - g_{m'}(l) - b_{mm'}(h) + b_{mm'}(l)|$,

2. $|g_{m'}(h) - g_{m'}(l) - b_{mm'}(h) + b_{mm'}(l)| > c \geq |g_m(h) - g_m(l) + b_{mm'}(h) - b_{mm'}(l)|$,

3. $|g_m(h) - g_m(l) + b_{mm'}(h) - b_{mm'}(l)|, |g_{m'}(h) - g_{m'}(l) - b_{mm'}(h) + b_{mm'}(l)| \leq c$,

4. $|g_m(h) - g_m(l) + b_{mm'}(h) - b_{mm'}(l)|, |g_{m'}(h) - g_{m'}(l) - b_{mm'}(h) + b_{mm'}(l)| > c$.

Case 1: Only m's incentive-compatibility constraint is violated in $g + b_{mm'}$. Without loss of generality, suppose that $g_m(l) - g_m(h) + b_{mm'}(l) - b_{mm'}(h) > c$. Since $g + b_{mm'} \succ_{m'} g$, it follows that

$$\pi u_{m'}(w_{m'}(H) + g_{m'}(h)) + (1 - \pi) u_{m'}(w_{m'}(L) + g_{m'}(l)) <$$
$$\pi u_{m'}(w_{m'}(H) + g_{m'}(l) - b_{mm'}(l)) + (1 - \pi) u_{m'}(w_{m'}(L) + g_{m'}(l) - b_{mm'}(l)).$$

Plugging constant absolute risk aversion into the above expression, we obtain inequality (1), which provides an upper bound for $b_{mm'}(l)$:

$$b_{mm'}(l) < \frac{1}{\gamma} log[\frac{\pi exp[\gamma(g_{m'}(l) - g_{m'}(h))] + (1 - \pi) exp[\gamma(w_{m'}(H) - w_{m'}(L))]}{\pi + (1 - \pi) exp[\gamma(w_{m'}(H) - w_{m'}(L))]}]. \quad (1)$$

We now find a consistent belief $\beta_m(b_{mm'}, g)$ to block $b_{mm'}$. Consider $\beta_m(b_{mm'}, g) = b_{m'k}$, such that $k \notin M$, $g_{m'}(l) - g_{m'}(h) + b_{m'k}(l) - b_{m'k}(h) > c$, and $g_{m'}(l) - g_{m'}(h) + b_{m'k}(l) - b_{m'k}(h) - b_{mm'}(l) + b_{mm'}(h) = c$. Under this belief, the side-contract $b_{mm'}$ has no effect on the signals' distribution (agent m''s incentive-compatibility constraint is violated under $g + b_{m'k}$ and agent m's incentive-compatibility constraint is violated under $g + b_{m'k} + b_{mm'}$). Rather, it affects the identity of the agent who pays for manipulation as agent m is the one who incurs the cost of manipulation under $g + b_{m'k} + b_{mm'}$. Observe that $g + b_{m'k} \not\succ_m g + b_{m'k} + b_{mm'}$ (i.e., the belief does not block the deviation) if and only if

$$\pi u_m(w_m(H) + g_m(l) + b_{mm'}(l) - c) + (1-\pi) u_m(w_m(L) + g_m(l) + b_{mm'}(l)) \geq \\ \pi u_m(w_m(H) + g_m(l)) + (1-\pi) u_m(w_m(L) + g_m(l)).$$

Plugging constant absolute risk aversion into the above expression, we obtain

$$b_{mm'}(l) \geq \frac{1}{\gamma} log[\frac{\pi exp[\gamma c] + (1-\pi) exp[\gamma(w_m(H) - w_m(L))]}{\pi + (1-\pi) exp[\gamma(w_m(H) - w_m(L))]}]. \tag{2}$$

Since g is IC, it follows that $c \geq g_{m'}(l) - g_{m'}(h)$ and, therefore, there exists no contract $b_{mm'}$ that satisfies both inequalities. It is left to verify that $\beta_m(b_{mm'}, g)$ is consistent with $b_{mm'}$. This is true if inequality (3) holds:

$$\pi u_{m'}(w_{m'}(H) + g_{m'}(l) + b_{m'k}(l) - c) + (1-\pi) u_{m'}(w_{m'}(L) + g_{m'}(l) + b_{m'k}(l)) \tag{3}$$
$$< \pi u_{m'}(w_{m'}(H) + g_{m'}(l) + b_{m'k}(l) - b_{mm'}(l)) \\ + (1-\pi) u_{m'}(w_{m'}(L) + g_{m'}(l) + b_{m'k}(l) - b_{mm'}(l)).$$

One can verify that inequality (3) is implied by inequality (1).

Case 2: Only m''s incentive-compatibility constraint is violated in $g + b_{mm'}$. Without loss of generality, suppose that $g_{m'}(l) - g_{m'}(h) - b_{mm'}(l) + b_{mm'}(h) > c$. Since $g + b_{mm'} \succ_{m'} g$, it must be that $b_{mm'}(l) < 0$. Consider $\beta_m(b_{mm'}, g) = b_{m'k}$, where $k \notin M$ and $g_{m'}(l) + b_{m'k}(l) - g_{m'}(h) - b_{m'k}(h) > c$. Observe that $PM(g + b_{m'k}) = PM(g + b_{m'k} + b_{mm'}) = \{m'\}$. Given this belief, the signal that results from both $g + b_{m'k}$ and $g + b_{m'k} + b_{mm'}$ is l (in both states) such that $b_{mm'}$ is simply a transfer from m to m'. Hence, m's belief is consistent with $b_{mm'}$ and blocks it.

Case 3: The contract $g+b_{mm'}$ is IC. Since $g+b_{mm'} \succ_{m'} g$, it must be that $b_{mm'}(h) < 0$ or $b_{mm'}(l) < 0$. Without loss of generality, assume that $b_{mm'}(h) < 0$ and consider a belief $\beta_m(b_{mm'}, g) = b_{m'k}$ such that $k \notin M$ and $g_{m'}(h) - g_{m'}(l) + b_{m'k}(h) - b_{m'k}(l) > c$. By Assumption 1, the signal that results from both $g + b_{m'k}$ and $g + b_{m'k} + b_{mm'}$ is h (in both states) such that $b_{mm'}$ is simply a transfer from m to m'. Hence, $\beta_m(b_{mm'}, g)$ is consistent with $b_{mm'}$ and blocks it.

Case 4: Both agents' incentive-compatibility constraints are violated under $g + b_{mm'}$. We shall show that $g + b_{mm'} \succ_{m'} g$ implies that $g \succ_m g + b_{mm'}$ such that $b_{mm'}$ is blocked by $\beta_m(b_{mm'}, g) = \emptyset$. Without loss of generality, suppose that $g_m(h) - g_m(l) + b_{mm'}(h) - b_{mm'}(l) > c$ and $g_{m'}(l) - g_{m'}(h) + b_{mm'}(h) - b_{mm'}(l) > c$. Observe that under g, each agent $d \in \{m, m'\}$ obtains a payoff greater than

$$\pi u_d(w_d(H) + min\{g_d(h), g_d(l)\}) + (1-\pi) u_d(w_d(L) + min\{g_d(h), g_d(l)\}). \quad (4)$$

By Assumption 2, under $g + b_{mm'}$, (i) the signal is independent of the state, (ii) agent m incurs a cost of c if $s = h$, and (iii) agent m' incurs a cost of c if $s = l$. By our additional assumption (the stronger Assumption 2), both signals are equally likely. Under $g + b_{mm'}$, agent m's payoff is as if, independently of the state, he were taking part in a lottery that pays with equal probability either $g_m(h) + b_{mm'}(h) - c$ or $g_m(l) + b_{mm'}(l)$. Agent m''s payoff is as if, independently of the state, he were taking part in a lottery that pays with equal probability either $g_{m'}(h) - b_{mm'}(h)$ or $g_{m'}(l) - b_{mm'}(l) - c$. Summing the agents' expected transfers and additional costs induced by these lotteries, we obtain

$$0.5(g_m(h) + g_m(l) + g_{m'}(h) + g_{m'}(l)) - c. \quad (5)$$

Since g is IC, (5) is at most $min\{g_m(h), g_m(l)\} + min\{g_{m'}(h), g_{m'}(l)\}$. This implies that if one of the two agents, m or m', is better off adding $b_{mm'}$ to g, then his counterparty is worse off signing this side-contract since his expected utility is strictly lower than (4). This is because adding $b_{mm'}$ guarantees an expected transfer that is independent of the state and lower than the minimal transfer induced by g. Thus, if $g + b_{mm'} \succ_{m'} g$, then $g \succ_m g + b_{mm'}$. ■

Lemmata 4 and 5 consider deviations by a pair of agents $i \notin M$ and $m \in M$. Lemma 4 shows a deviation that cannot be blocked by a consistent belief. Lemma 5 establishes that the deviation presented in Lemma 4 is the only deviation we need to consider when checking the weak stability of an IC multilateral contract.

Lemma 4 *Let g be IC and let $i \in I - M, m \in M$. If $g + b_{mi} \succ_i g$, $b_{mi}(h) > 0$, and $b_{mi}(l) > 0$, then there exists no consistent belief $\beta_m(b_{mi}, g)$ such that $g + \beta_m(b_{mi}, g) \succ_m g + \beta_m(b_{mi}, g) + b_{mi}$.*

Proof. Observe that since $g + b_{mi} \succ_i g$, $b_{mi}(h) > 0$, and $b_{mi}(l) > 0$, it must be that $g + b_{mi}$ is not IC. Since g is IC and $i \notin M$, it follows that $PM(g + b_{mi}) = \{m\}$. By Assumption 1, either $g + b_{mi}$ results in the signal h in both states or it results in the signal l in both states. Without loss of generality, assume that it results in the signal h in both states.

If $g+\beta_m(b_{mi}, g)$ is IC, then it cannot block b_{mi} as b_{mi} induces a positive transfer from i to m in both realisations. If $g + \beta_m(b_{mi}, g)$ is not IC, then $|PM(g + \beta_m(b_{mi}, g))| = 1$ and either it results in the signal h in both states or it results in the signal l in both states. If $g + \beta_m(b_{mi}, g)$ results in the signal l in both states, then m is better off signing b_{mi} as he obtains either $g_m(l) + b_{mi}(l) > g_m(l)$ or $g_m(h) + b_{mi}(h) - c > g_m(l)$ under $g + b_{mi} + \beta_m(b_{mi}, g)$ (the inequality $g_m(h) + b_{mi}(h) - c > g_m(l)$ follows from the fact that $PM(g + b_{mi}) = \{m\}$ and $b_{mi}(h) > b_{mi}(l) > 0$). If $g + \beta_m(b_{mi}, g)$ results in the signal h in both states, then, by Assumption 2, $g + b_{mi} + \beta_m(b_{mi}, g)$ results in the signal h in both states and, given $\beta_m(b_{mi}, g)$, b_{mi} is a positive transfer from i to m. Such a belief is inconsistent with b_{mi} as i pays m without the payment affecting the realisation of the signal ex post. ■

Lemma 5 *Let g be IC, let $i \in I - M, m \in M$, and suppose that there exists no side-contract b_{mi} such that $g + b_{mi} \succ_i g$, $b_{mi}(h) > 0$, and $b_{mi}(l) > 0$. For each side-contract b_{mi} such that $g + b_{mi} \succ_i g$ and $g + b_{mi} \succ_m g$, there exist consistent beliefs $\beta_i(b_{mi}, g)$ and $\beta_m(b_{mi}, g)$ such that $g + \beta_i(b_{mi}, g) \succ_i g + \beta_i(b_{mi}, g) + b_{mi}$ and $g + \beta_m(b_{mi}, g) \succ_m g + \beta_m(b_{mi}, g) + b_{mi}$.*

Proof. Let g be IC and let b_{mi} be an arbitrary side-contract such that $g + b_{mi} \succ_i g$ and $g + b_{mi} \succ_m g$. We shall now find two beliefs $\beta_i(b_{mi}, g)$ and $\beta_m(b_{mi}, g)$ that are consistent with b_{mi} and that block it. Observe that if both $b_{mi}(h) \leq 0$ and $b_{mi}(l) \leq 0$, then $g + b_{mi} \not\succ_m g$. By the assumption in the premise, it follows that either $b_{mi}(h) > 0 > b_{mi}(l)$ or $b_{mi}(l) > 0 > b_{mi}(h)$. Without loss of generality, let $b_{mi}(h) > 0 > b_{mi}(l)$. There are two possible cases: either $g + b_{mi}$ is IC or not.

Suppose that $g + b_{mi}$ is IC. Consider agent i and a belief $\beta_i(b_{mi}, g) = b_{mk}$ such that $k \notin M$ and $g_m(h) - g_m(l) + b_{mk}(h) - b_{mk}(l) > c$. Note that $PM(g + b_{mk}) = PM(g + b_{mk} + b_{mi}) = \{m\}$. By Assumption 1, the contracts $g + b_{mi} + b_{mk}$ and $g + b_{mk}$ result in the signal h (in both states). Hence, given b_{mk}, the contract b_{mi} is a positive transfer from i to m in both states. Thus, $\beta_i(b_{mi}, g) = b_{mk}$ is consistent and blocks b_{mi}. Consider agent m and a belief $\beta_m(b_{mi}, g) = b_{ki}$ such that $k \in M$ and $g_k(l) - g_k(h) + b_{ki}(l) - b_{ki}(h) > c$. Note that $PM(g + b_{ki}) = PM(g + b_{ki} + b_{mi}) = \{k\}$. By Assumption 1, the signal resulting from $g + b_{mi} + b_{mk}$ and from $g + b_{mk}$ is l (in both states) such that, given $\beta_m(b_{mi}, g) = b_{ki}$, b_{mi} is a positive transfer from m to i in both states. Thus, $\beta_m(b_{mi}, g) = b_{ki}$ is consistent and blocks b_{mi}.

Let us assume that $g+b_{mi}$ is not IC. Then, $PM(g + b_{mi}) = \{m\}$. Since $g+b_{mi} \succ_i g$ and $b_{mi}(h) > b_{mi}(l)$, it follows that $g_i(h) > g_i(l)$. First, we find a consistent belief $\beta_i(b_{mi}, g)$ that blocks b_{mi}. Consider $\beta_i(b_{mi}, g) = b_{mk}$ such that $k \notin M$ and $g_m(h) - g_m(l) + b_{mk}(h) - b_{mk}(l) > c$. Observe that $PM(g + b_{mk}) = PM(g + b_{mk} + b_{mi}) = \{m\}$. By Assumption 1, both contracts result in the signal h in both states. Since under this belief b_{mi} is a transfer from i to m that does not affect the realisation of the signal, it follows that $\beta_i(b_{mi}, g) = b_{mk}$ is consistent with b_{mi} and $g+b_{mk} \succ_i g+b_{mk}+b_{mi}$. It is left to construct a consistent belief $\beta_m(b_{mi}, g)$ that blocks b_{mi}.

Let $\beta_m(b_{mi}, g) = b_{ki}$ such that $k \in M$ and $g_k(l) - g_k(h) + b_{ki}(l) - b_{ki}(h) > c$. Furthermore, assume that $b_{ki}(l) > 0 > b_{ki}(h)$. Note that $PM(g + b_{ki}) = \{k\}$ and $PM(g + b_{ki} + b_{mi}) = \{k, m\}$. By Assumption 1, $s = l$ in both states under $g + b_{ki}$ such that m's transfer is $g_m(l)$ regardless of the state. By Assumption 2, given the multilateral contract $g + b_{mi} + b_{ki}$, m's transfer minus the cost he incurs is either $g_m(l) + b_{mi}(l) < g_m(l)$ (in the case of $s = l$) or $g_m(h) + b_{mi}(h) - c$ (in the case of $s = h$). We now show that $g_m(h) + b_{mi}(h) - c < g_m(l)$.

Since $g + b_{mi} \succ_i g$, it must be that

$$\begin{aligned} \pi_i u_i(w_i(H) + g_i(h)) + (1 - \pi_i) u_i(w_i(L) + g_i(l)) < \\ \pi_i u_i(w_i(H) + g_i(h) - b_{mi}(h)) + (1 - \pi_i) u_i(w_i(L) + g_i(h) - b_{mi}(h)). \end{aligned} \tag{6}$$

If we add a contract $\hat{b}_{mi}$ to g such that $PM\left(g + \hat{b}_{mi}\right) = \{m\}$ and $\hat{b}_{mi}(h) = b_{mi}(h) > \hat{b}_{mi}(l)$, then agent i's expected payoff is as described on the RHS of (6). By the assumption (made in the premise) that there exists no contract $\hat{b}_{mi}$ such that $\hat{b}_{mi}(h) > \hat{b}_{mi}(l) > 0$ and $g+\hat{b}_{mi} \succ_i g$, it must be that $g_m(h) - g_m(l) + b_{mi}(h) < c$. That is, it must be that m's incentive-compatibility constraint is not violated by adding a contract $\hat{b}_{mi}$ to g such that $\hat{b}_{mi}(h) = b_{mi}(h) > \hat{b}_{mi}(l) \geq 0$. It follows that $g_m(h)+b_{mi}(h)-c < g_m(l)$. Therefore, $g + b_{ki} \succ_m g + b_{ki} + b_{mi}$.

We now show that the belief $\beta_m(b_{mi}, g)$ is consistent with b_{mi}. Agent i's expected payoff in $g + b_{ki}$ is

$$\pi_i u_i\left(w_i\left(H\right) + g_i\left(l\right) - b_{ki}\left(l\right)\right) + \left(1-\pi_i\right) u_i\left(w_i\left(L\right) + g_i\left(l\right) - b_{ki}\left(l\right)\right).$$

This expression is less than the LHS of (6) since $b_{ki}\left(l\right) > 0$ and $g_i\left(h\right) > g_i\left(l\right)$. Under $g + b_{ki} + b_{mi}$ agent i obtains a payoff of

$$\pi_i u_i\left(w_i\left(H\right) + g_i\left(h\right) - b_{mi}\left(h\right) - b_{ki}\left(h\right)\right) + \left(1-\pi_i\right) u_i\left(w_i\left(L\right) + g_i\left(h\right) - b_{mi}\left(h\right) - b_{ki}\left(h\right)\right)$$

if the signal is h. This expression is greater than the RHS of (6) since $b_{ki}\left(h\right) < 0$. Moreover, under $g + b_{ki} + b_{mi}$ agent i obtains a payoff of

$$\pi_i u_i\left(w_i\left(H\right) + g_i\left(l\right) - b_{ki}\left(l\right) - b_{mi}\left(l\right)\right) + \left(1-\pi_i\right) u_i\left(w_i\left(L\right) + g_i\left(l\right)\right) - b_{ki}\left(l\right) - b_{mi}\left(l\right)\right)$$

if the signal is l. This expression is greater than i's payoff under $g + b_{ki}$ as $b_{mi}\left(l\right) < 0$. Thus, in every realization of the signal, agent i's expected payoff under $g + b_{mi} + b_{ki}$ is greater than his payoff under $g + b_{ki}$. Hence, m's belief is consistent. ■

By the four lemmata, a *necessary and sufficient condition* for the weak stability of an IC multilateral contract is that there exists no side-contract b_{mi} such that $i \notin M$, $m \in M$, $b_{mi}\left(h\right) \geq 0$, $b_{mi}\left(l\right) \geq 0$, and $g + b_{mi} \succ_i g$.

For the aforementioned side-contract b_{mi} to exist, there must be an agent $i \notin M$ who is willing to pay $b = max\left\{b_{mi}\left(h\right), b_{mi}\left(l\right)\right\}$ to guarantee his preferred realisation $s \in \{l, h\}$, and an agent $m \in M$ such that $g_m\left(s\right) - g_m\left(s'\right) + b > c$. Thus, weak stability can be written as two constraints (one for each realisation of the signal), which are given in (7) and (8). For each $i \notin M$, denote i's ex-ante willingness to pay to guarantee a realisation of s ex post by $z_i\left(g, s\right)$:

$$max_{i \notin M} z_i\left(g, h\right) + max_{m \in M}\left\{g_m\left(h\right) - g_m\left(l\right)\right\} \leq c, \tag{7}$$

$$max_{i \notin M} z_i\left(g, l\right) + max_{m \in M}\left\{g_m\left(l\right) - g_m\left(h\right)\right\} \leq c. \tag{8}$$

Plugging CARA into z_i, for each $i \in L - M$ we get

$$z_i\left(g, l\right) = \frac{1}{\gamma} log\left[\frac{\pi exp\left[\gamma\left(g_i\left(l\right) - g_i\left(h\right)\right)\right] + \left(1-\pi\right) exp\left[\gamma\left(w_i\left(H\right) - w_i\left(L\right)\right)\right]}{\pi + \left(1-\pi\right) exp\left[\gamma\left(w_i\left(H\right) - w_i\left(L\right)\right)\right]}\right]$$

while for each $i \in E$, $z_i\left(g, l\right) = \pi\left(g_i\left(l\right) - g_i\left(h\right)\right)$. Note that $z_i\left(g, h\right)$ is symmetric. Agent i's willingness to pay to guarantee his preferred realisation is pinned down by $g_i\left(h\right) - g_i\left(l\right)$ (and it is monotone in $g_i\left(h\right) - g_i\left(l\right)$). It follows that for every IR, IC, and weakly stable contract g, there exists an IR and IC multilateral contract $\hat{g}$ that provides the same total coverage, satisfies (7) and (8), and for which, for each pair of agents $i, j \in I \in \{M, L-M, E\}$, it holds that $\hat{g}_i(h) - \hat{g}_i(l) = \hat{g}_j(h) - \hat{g}_j(l)$. We shall refer to such contracts as *symmetric* contracts. For every symmetric contract g, denote by $R_m(g) := g_m(l) - g_m(h)$, $R_k(g) := g_k(l) - g_k(h)$, and $R_e(g) := g_e(l) - g_e(h)$ the coverage provided to each $m \in M$, $k \in L-M$, and $e \in E$, respectively. Without loss of generality, we shall restrict attention to symmetric multilateral contracts.

Observe that $R_m \leq c$ in every IC multilateral contract. If $R_m \leq c$, then (8) is not violated by $R_k = 0$. Hence, every multilateral contract that maximises the total coverage within the class of IC and weakly stable multilateral contracts must have $R_k \geq 0$.

We now show that the weak-stability constraint (8) must be binding at the optimum. Inequality (8) can be written as

$$R_m(g) \leq c - \frac{1}{\gamma} log[\frac{\pi exp[\gamma R_k(g)] + (1-\pi) exp[\gamma w]}{\pi + (1-\pi) exp[\gamma w]}]. \tag{9}$$

If (9) is not binding, then we can add to g a multilateral contract g' in which the members of E provide the members of M with coverage (i.e., a contract that increases R_m), without violating IC, IR, and (9). Since $c|E| > w|L|$, we can split the contract equally between the members of E such that the second weak-stability constraint given in (7) is not violated by this addition either. Thus, if g maximises the coverage under the IR, IC, and weak-stability constraints, then (9) must hold with equality.

Observe that (9) is independent of α. For each $\alpha \in [0,1]$, define R_m^α, R_k^α, and R_e^α, respectively, to be the values of R_m, R_k, and R_e that maximise the total coverage that can be provided in a weakly stable, IC, and IR contract given α. Ignoring (7) and IR for a moment, finding the maximal total coverage is equivalent to maximising a convex combination of $\alpha R_m + (1-\alpha) R_k$ subject to (9) (which is concave), subject to the IC constraints that $R_m^\alpha \in [-c, c]$, and subject to the constraint that $R_k^\alpha \in [0, w]$. This implies that R_m^α is weakly increasing in α, and that there exists an $\alpha^\star \in (0,1)$ such that $R_m^{\alpha^\star} = R_k^{\alpha^\star} > 0$ and $\alpha R_m^\alpha + (1-\alpha) R_k^\alpha$ is strictly increasing (respectively, decreasing) in α for $\alpha > \alpha^\star$ (respectively, $\alpha < \alpha^\star$).

As long as $R_m^\alpha \geq 0$, the IR constraints hold as the risk-neutral reinsurers provide coverage to the risk-averse insurers. Moreover, the second weak-stability constraint in (7) holds when $R_m^\alpha \geq 0$ since it is possible to split the coverage such that $R_e^\alpha > -c$ and the external reinsurers are unwilling to pay more than c to guarantee the signal h ex post. Thus, the total coverage is increasing in α in $[\alpha^\star, 1]$. It is left to show that the total coverage is decreasing in α when $\alpha < \alpha^{\star}$.

For $\alpha < \alpha^{\star}$, it holds that $R_m^{\alpha} < R_k^{\alpha}$. Let us consider two values of α: $\alpha' < \alpha'' < \alpha^{\star}$ and show that $\alpha' R_m^{\alpha'} + (1-\alpha') R_k^{\alpha'} > \alpha'' R_m^{\alpha''} + (1-\alpha'') R_k^{\alpha''}$. Consider an IR, IC, and weakly stable contract g that provides coverage of $R_m^{\alpha''}$ and $R_k^{\alpha''}$ when $\alpha = \alpha''$. Let us decrease the share of manipulators from α'' to α'. Denote by M_1 the set of agents who are nonmanipulators under α' and manipulators under α''. We can add to g the contract g' in which the members of E provide more coverage to the members of M_1 such that $g_i(l) - g_i(h) + g_i'(l) - g_i'(h) = R_k^{\alpha''} > R_m^{\alpha''}$. As the risk-neutral reinsurers provide coverage to the risk-averse insurers, g' can be set such that $g + g'$ does not violate the IR constraints. The weak-stability constraints in (9) and (7) are not violated by this change either as $R_e \geq -c$ after the change, and the coverage provided to manipulators and nonmanipulators remains as before. Let T be the total coverage in $g + g'$ after the change and observe that $\alpha' R_m^{\alpha'} + (1-\alpha') R_k^{\alpha'} \geq T$. Since $R_k^{\alpha''} > R_m^{\alpha''}$, it follows that $T > \alpha'' R_m^{\alpha''} + (1-\alpha'') R_k^{\alpha''}$.

Proof of Proposition 7

By the incentive-compatibility property, $g_m(h) - g_m(l) \in [-c, c]$ for every $m \in M$. Consider an arbitrary agent $i \notin M$ and assume that $g_i(h) > g_i(l)$. Ex ante, agent i is willing to pay $(g_i(h) - g_i(l))(1-\pi_i)$ to guarantee that, ex post, the realisation will be h. We now show that if g is IC and $(g_i(h) - g_i(l))(1-\pi_i) > 2c$, then g is not weakly stable.

Consider a side-contract b_{mi} such that $b_{mi}(h) = 2c + 2\epsilon > b_{mi}(l) = \epsilon > 0$. As g is IC, $PM(g + b_{ij}) = \{m\}$. Since $g_m(h) - g_m(l) + b_{mi}(h) - b_{mi}(l) > c$, by Assumption 1, the signal h results from $g + b_{mi}$ in both states. Thus, if $\epsilon > 0$ is sufficiently small, $g + b_{mi} \succ_i g$. Since m receives a positive transfer in both realisations and g is IC, it must be that $g + b_{mi} \succ_m g$.

We now show that there exists no consistent belief $\beta_m(b_{mi}, g)$ that blocks b_{mi}. Clearly, if $g + \beta_m(b_{mi}, g)$ is IC, then $g + \beta_m(b_{mi}, g) + b_{mi} \succ_m g + \beta_m(b_{mi}, g)$. Suppose that $g + \beta_m(b_{mi}, g)$ is not IC. Then , since $i \notin M$ and g is IC, $|PM(g + \beta_m(b_{mi}, g))| = 1$. By Assumption 1, either $g + \beta_m(b_{mi}, g)$ results in l in both states or it results in h in both states. If $g + \beta_m(b_{mi}, g)$ results in l in both states, then m is better off signing b_{mi} as it guarantees him a transfer greater than $g_m(l)$. If $g + \beta_m(b_{mi}, g)$ results in h in both states, then, by Assumption 2, $g + \beta_m(b_{mi}, g) + b_{mi}$ results in h in both states such that b_{mi} is a transfer from i to m that does not affect the signal. This is in contradiction to the consistency of $\beta_m(b_{mi}, g)$. In conclusion, if g is IC and weakly stable, then $(g_i(h) - g_i(l))(1 - \pi_i) \leq 2c$ for every $i \in I - M$. A symmetric argument shows that $g_i(l) - g_i(h)$ is bounded from above for every $i \in I - M$.

Proof of Proposition 8

First, we need to determine which deviations can undermine the weak stability of a multilateral contract, and which deviations cannot do so. Lemmata 6–9 show that, as in the proof of Proposition 6, the deviations that we need to look for consist of a side-contract between an agent $i \notin M$ and an agent $m \in M$. In these side-contracts, agent i pays agent m in order to incentivise the latter to manipulate the signal to i's preferred realisation ex post, if necessary.

The analysis in this part of the proof is similar to that in the proof of Proposition 6 and we shall refer the reader to the proof of Proposition 6 whenever there is redundancy.

Lemma 6 *Let g be IC and IR and let $i, j \in I - M$. For each side-contract b_{ij} such that $g + b_{ij} \succ_i g$, there exists a consistent belief $\beta_j(b_{ij}, g)$ such that $g + \beta_j(b_{ij}, g) \succ_j g + \beta_j(b_{ij}, g) + b_{ij}$.*

Proof. See the proof of Lemma 2. ■

Lemma 7 *Let g be an IC and IR contract that satisfies Condition 1. Let $m, m' \in M$. For each side-contract $b_{mm'}$ such that $g + b_{mm'} \succ_{m'} g$, there exists a consistent belief $\beta_m(b_{mm'}, g)$ such that $g + \beta_m(b_{mm'}, g) \succ_m g + \beta_m(b_{mm'}, g) + b_{mm'}$.*

Proof. There are four possible cases:

1. $|g_m(h) - g_m(l) + b_{mm'}(h) - b_{mm'}(l)| > c \geq |g_{m'}(h) - g_{m'}(l) - b_{mm'}(h) + b_{mm'}(l)|$,
2. $|g_{m'}(h) - g_{m'}(l) - b_{mm'}(h) + b_{mm'}(l)| > c \geq |g_m(h) - g_m(l) + b_{mm'}(h) - b_{mm'}(l)|$,
3. $|g_m(h) - g_m(l) + b_{mm'}(h) - b_{mm'}(l)|, |g_{m'}(h) - g_{m'}(l) - b_{mm'}(h) + b_{mm'}(l)| \leq c$,
4. $|g_m(h) - g_m(l) + b_{mm'}(h) - b_{mm'}(l)|, |g_{m'}(h) - g_{m'}(l) - b_{mm'}(h) + b_{mm'}(l)| > c$.

Case 1: Only m's incentive-compatibility constraint is violated under $g + b_{mm'}$. First, observe that if $sign(g_m(h) - g_m(l)) \neq sign(g_{m'}(h) - g_{m'}(l))$ and only m's incentive-compatibility constraint is violated, then it cannot be that both $g \not\succ_m g + b_{mm'}$ and $g + b_{mm'} \succ_{m'} g$. Hence, in this case $g \succ_m g + b_{mm'}$ such that a belief $\beta_m(b_{mm'}, g) = \emptyset$ is consistent and blocks $b_{mm'}$.

Second, assume that $sign(g_m(h) - g_m(l)) = sign(g_{m'}(h) - g_{m'}(l))$. By Condition 1, the side-contract is signed between an agent $m \in I^l \cap M$ and an agent $m' \in I^l \cap M$

or between an agent $m \in I^h \cap M$ and an agent $m' \in I^h \cap M$. If we plug risk-neutral preferences into (1), (2), and (3) instead of CARA preferences in Case 1 of Lemma 3, then we obtain that a side-contract between an agent $m \in I^l \cap M$ and an agent $m' \in I^l \cap M$ such that only agent m's incentive-compatibility constraint is violated cannot destabilise a multilateral contract. Analogously, a side-contract between an agent $m \in I^h \cap M$ and an agent $m' \in I^h \cap M$ such that only m's incentive-compatibility constraint is violated cannot destabilise a multilateral contract.

Case 2: Only m''s incentive-compatibility constraint is violated in $g + b_{mm'}$. The proof is identical to that of Case 2 in Lemma 3.

Case 3: The contract $g + b_{mm'}$ is IC. The proof is identical to that of Case 3 in Lemma 3.

Case 4: Both agents' incentive-compatibility constraints are violated under $g + b_{mm'}$. The proof is identical to that of Case 4 in Lemma 3.

Lemma 8 *Let g be IC and IR and let $i \in I - M, m \in M$. For each side-contract b_{mi} such that $g + b_{mi} \succ_i g$, $b_{mi}(h) > 0$, and $b_{mi}(l) > 0$, there exists no consistent belief $\beta_m(b_{mi}, g)$ such that $g + \beta_m(b_{mi}, g) \succ_m g + \beta_m(b_{mi}, g) + b_{mi}$.*

Proof. The proof of this lemma is identical to the proof of Lemma 4. ■

Lemma 9 *Let g be IC and IR, let $i \in I - M, m \in M$, and suppose that there exists no side-contract b_{mi} such that $g + b_{mi} \succ_i g$, $b_{mi}(h) > 0$, and $b_{mi}(l) > 0$. For each contract b_{mi} such that $g + b_{mi} \succ_i g$ and $g + b_{mi} \succ_m g$, there exist consistent beliefs $\beta_i(b_{mi}, g)$ and $\beta_i(b_{mi}, g)$ such that $g + \beta_i(b_{mi}, g) \succ_i g + \beta_i(b_{mi}, g) + b_{mi}$ and $g + \beta_m(b_{mi}, g) \succ_m g + \beta_m(b_{mi}, g) + b_{mi}$.*

Proof. The proof is identical to the proof of Lemma 5. ■

By Lemmata 6–9, to verify the weak stability of an IC multilateral contract g that satisfies Condition 1, we only need to check whether there exists a contract b_{mi} such that $i \notin M$, $m \in M$, $b_{mi}(h) \geq 0$, $b_{mi}(l) \geq 0$, and $g + b_{mi} \succ_i g$. For such a contract to exist, there must be an agent $i \notin M$ who is willing to pay $b = max\{b_{mi}(h), b_{mi}(l)\}$ to guarantee his preferred signal $s \in \{l, h\}$, and an agent $m \in M$ such that $g_m(s) - g_m(s') + b > c$. Condition 1 allows us to translate this to two "weak-stability constraints," one for the members of I^h and the other for the members of I^l:

$$max_{i \in I^h - M}(g_i(h) - g_i(l))(1 - \pi_h) \leq c - max_{m \in M \cap I^h}(g_m(h) - g_m(l)), \tag{10}$$

$$max_{i \in I^l - M}(g_i(l) - g_i(h))\pi_l \leq c - max_{m \in M \cap I^l}(g_m(l) - g_m(h)). \tag{11}$$

The LHS of (10) (respectively, (11)) is the maximal willingness of any agent $i \in I^h - M$ (respectively, $i \in I^l - M$) to pay ex ante to guarantee that ex post the signal's realisation will be $s = h$ (respectively, $s = l$).

In addition to weak stability, the multilateral contract has to be IC and IR. The multilateral contract's incentive compatibility is implied by the combination of Condition 1 and the constraints in (10) and (11). Observe that the total surplus is

$$\sum_{i \in I} \pi_i g_i(h) + (1 - \pi_i) g_i(l) = \sum_{i \in I} \pi_i (g_i(h) - g_i(l)). \tag{12}$$

Condition 1 implies that (12) is positive. Hence, even if g is not IR, there exists another IR contract g' such that $g_i(h) - g_i(l) = g'_i(h) - g'_i(l)$ for each $i \in I$. In conclusion, when we look for the maximal volume of speculative trade that is induced by an IC, IR, and weakly stable contract that satisfies Condition 1, we can restrict our attention to multilateral contracts that satisfy Condition 1, (10), and (11).

Let us consider constraint (10) and the members of I^h. Each $i \in I^h - M$ can hold a position of $g_i(h) - g_i(l) \leq min_{m \in I^h \cap M} \frac{c - (g_m(h) - g_m(l))}{1 - \pi_h}$. Each $m \in I^h \cap M$ can hold a position of $g_m(h) - g_m(l) \leq min_{i \in I^h - M} c - (1 - \pi_h)(g_i(h) - g_i(l))$. Thus, the maximal volume of speculative trade subject to (10) is obtained by a contract g in which each $m \in I^h \cap M$ holds a position of $g_m(h) - g_m(l) = c$ or by a contract g' in which each $i \in I^h - M$ holds a position of $g'_i(h) - g'_i(l) = \frac{c}{1 - \pi_h}$. Hence, the maximal volume of speculative trade subject to (10) is $n * max \left\{ \alpha c, (1 - \alpha) \frac{c}{1 - \pi_h} \right\}$. A symmetric analysis shows that the maximal volume of speculative trade subject to (11) is $n * max \left\{ \alpha c, (1 - \alpha) \frac{c}{\pi_l} \right\}$. Since the two groups of agents bet against each other, the maximal volume of speculative trade is the minimum of the two expressions. ■

Appendix B: The Maximal Aggregate Coverage in Section 4.1

First, let us plug the weak-stability constraint given in (9) into our objective function, $\alpha R_m + (1 - \alpha) R_k$, to obtain (13):

$$\alpha \left(c - \frac{1}{\gamma} log[\frac{\pi exp[\gamma R_k] + (1 - \pi) exp[\gamma w]}{\pi + (1 - \pi) exp[\gamma w]}] \right) + (1 - \alpha) R_k. \tag{13}$$

We now maximise (13) under the IR and IC constraints, the additional weak-stability constraint given in (7), and the restriction to $R_k \leq w$. Note that if $R_m \in [0, c]$ and $R_k \in [0, w]$ in some contract g that is not IR, then we can always find another contract g' that is both IR and satisfies the weak-stability constraint (7), and provides a coverage of R_m and R_k. Hence, in an internal solution, IR is satisfied. Moreover, in such a solution, IC is implied by the weak-stability constraint given in (9). From the first-order condition we obtain that in an internal solution,

$$R_k = w - \frac{1}{\gamma} log[\frac{\pi\,(2\alpha - 1)}{(1-\pi)\,(1-\alpha)}]. \tag{14}$$

The coverage R_k is decreasing in π since the willingness to pay to guarantee that the signal is l of each $k \in L - M$ is increasing in the probability that state H is realised. This follows from the fact that $k \in L - M$ benefits from manipulation only when state H is realised. Let us focus on the share of manipulators α. Intuitively, since we maximise a convex combination of R_k and R_m, it follows that when α is increasing, R_k is decreasing and R_m is increasing.

In an internal solution,

$$R_m = \left(c - \frac{1}{\gamma} log[\frac{\pi exp[\gamma\left(w - \frac{1}{\gamma} log[\frac{\pi(2\alpha-1)}{(1-\pi)(1-\alpha)}]\right)] + (1-\pi)\, exp[\gamma w]}{\pi + (1-\pi)\, exp[\gamma w]}] \right). \tag{15}$$

Hence the maximal coverage per insurer that can be obtained in an IR, IC, and weakly stable contract is

$$\alpha \left(c - \frac{1}{\gamma} log[\frac{\pi exp[\gamma\,(w-\Delta)] + (1-\pi)\, exp[\gamma w]}{\pi + (1-\pi)\, exp[\gamma w]}] \right) + (1-\alpha)\,(w - \Delta)], \tag{16}$$

where $\Delta := \frac{1}{\gamma} log[\frac{\pi(2\alpha-1)}{(1-\pi)(1-\alpha)}]$. To verify that the solution is indeed internal we must verify that two conditions hold. First, it must be that $R_k \in [0, w]$. Observe that this condition is satisfied for $\alpha \in [\frac{1}{1+\pi}, \frac{\pi+(1-\pi)exp[\gamma w]}{2\pi+(1-\pi)exp[\gamma w]}]$. The second condition needed for an internal solution is that if $R_m \in (-c, 0)$ (that is, if the members of M "have skin in the game"), then both IR and the second weak-stability constraint, explicitly given in (17), hold:

$$c + R_m \geq -\,(1-\pi)\, R_e. \tag{17}$$

Let us consider the following parameters: $w = 10, \gamma = 0.5, \pi = 0.9$, and $c = 8$. For $\alpha \in [0.526, 0.945]$ we obtain an internal solution for R_k. For $\alpha < 0.526$ ($\alpha > 0.945$), $R_k = 10$ ($R_k = 0$) and $R_m = 3.512$ ($R_m = 8$). For $\alpha = 0.75$, the maximal average level of coverage is 6.534, which reflects a loss of 23.13 percent when compared to the constrained-efficient level of coverage, which is 8.5. If we increase the agents' risk aversion to $\gamma = 1$, we have an internal solution to R_k for $\alpha \in [0.526, 0.999]$. For

$\alpha < 0.526$ ($\alpha > 0.999$), we have $R_k = 10$ ($R_k = 0$) and $R_m = 5.697$ ($R_m = 8$).