Multilateral Contracting with Manipulation*

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Abstract

We study multilateral risk-sharing when the state of nature is unverifiable, so that contracts are conditioned on a state-dependent signal (e.g., net earnings in a financial report). A subset of the agents can manipulate the signal's realisation at some cost and as a result Pareto-optimal reallocation of risk is precluded. The agents can write additional side-contracts that can be used to incentivise one of the parties to manipulate the signal. Using a novel stability notion that takes into account agents' beliefs about contemporaneous deviations initiated by their counterparties, we explore the limits of risk-sharing and risk-bearing.

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1 Introduction

It is well known that when agents have access to Arrow–Debreu securities, they can reallocate risk efficiently. In practice, however, state-contingent contracts are not always feasible, as the state of nature may be unobservable, unverifiable, or hard to assess to the point where state-contingent contracts are unenforceable or too costly to implement.

For these reasons, risk-sharing contracts are often contingent on verifiable variables that are informative about the state. For example, financial benchmarks such as the inter-bank offered rates "have been heavily used in contracts whose purpose is to transfer risk related to fluctuations in general market-wide interest rates" (Duffie and Stein, 2015). Other prominent examples are insurance contracts, which are often contingent on an appraisal rather than on actual damage, catastrophe bonds (e.g., World Bank Pandemic Bonds), which are often contingent on indices, evaluations, and statements by official authorities rather than on actual damages, and managerial compensation contracts, which are often contingent on a firm's net earnings as they appear in its financial reports rather than on the firm's actual performance. In the present paper, we focus on these types of contracts and refer to the contractible variable as a *signal*.

Transferring risk by means of signal-contingent contracts gives rise to a *moral hazard* problem that results from the agents' ability to *manipulate* the signal by taking costly actions. Such costly actions include: forging an appraisal or misreporting the occurrence of an insurable event; deferring recognition of some expenditure to change a firm's net earnings on a specific date; inflating future prices in a commodity market by placing large buy orders in the underlying market; and hiring lobbyists to influence a policy on which a contract depends.

In this paper, we examine how the above moral hazard affects individuals' ability to share risk. To this end, we study a model in which multiple agents transfer risk by means of contracts that are budget-balanced transfers contingent on a signal. We assume that a subset of agents can manipulate the signal unilaterally by incurring some cost. When for each agent the cost of manipulating the signal is greater than the corresponding benefit, the collection of contracts is said to be incentive compatible (IC) and there is no manipulation. In such cases, the signal perfectly reveals the state. To illustrate some of the model's features, we present the following example.

Example 1 *There are two states, high (H) and low (L). Alice and Bob are each exposed to a negative shock of* 100 *dollars in state L and a risk-neutral insurer is willing to share some of the risk for a premium. Risk-sharing contracts are contingent* *on an appraisal* $s \in \{h, l\}$ *made by a certified appraiser. The appraiser reports* h *in state* H *and* l *in state* L*. Alice and Bob both know the appraiser and, in state* H*, each of them can pay him a bribe of* 90 *dollars to change his appraisal from* h *to* l*. Observe that full insurance is not IC as it incentivises Alice and Bob to bribe the appraiser in state* H*. Because of the moral hazard, each of them can receive a coverage of at most* 90 *dollars. We shall refer to such coverage as constrained-efficient coverage.*

A key feature in this work is that, at the contracting stage, before the state is realised, agents can add new contracts to the existing collection of contracts. We refer to these contracts as *side-contracts.* A side-contract can be used to provide legitimate mutual insurance or to *incentivise* one of the contracting parties to manipulate the signal ex post. The latter type of side-contract introduces a new source of instability into multilateral risk-sharing since it imposes an externality on third parties.

Using Example 1, let us demonstrate how a pair of agents can benefit from a side-contract that incentivises one of them to manipulate the signal ex post. Suppose that Alice and Bob each receive a coverage of 90 dollars. At the contracting stage, Bob and Alice can benefit from adding a side-contract in which Bob pays Alice a small $\epsilon > 0$ if and only if $s = l$. This side-contract incentivises Alice to bribe the appraiser in state H, which makes Bob better off: he guarantees his preferred appraisal by paying a small cost of ϵ. Alice is also better off since she obtains an extra ϵ from Bob. We can conclude that the side-contract makes both agents better off when the possibility of ex-post manipulation is taken into account.

Side-contracts can have a negative effect on a third party due to the contracting parties' ability to manipulate the signal ex post. For instance, the ex-post manipulation of the appraisal imposes a negative externality on an insurer who provides Alice and Bob with coverage. In fact, an insurer who predicts the side-contract between Alice and Bob would not provide them with coverage due to this negative effect. This type of *contractual externality* plays a key role in our model.

Since contracts require mutual consent, we study collections of contracts that are robust to both unilateral and multilateral deviations. We say that a collection of contracts is stable if no group of agents are better off writing a new side-contract and no agent is better off cancelling a previously signed contract unilaterally. This cooperative notion is inspired by the network formation literature (which typically focuses on single and pairwise deviations; see, e.g., Jackson and Wolinsky, 1996) and it enables us to refrain from making particular assumptions about the way contracts are negotiated.

We illustrate that IC stable collections of contracts do not exist when manipulation is not prohibitively costly. The reason for this effect is that our stability notion considers one deviation at a time, which is an implicit assumption that whenever a side-contract is beneficial for the deviating parties, they will sign it.[1] This assumption is particularly restrictive when there are contractual externalities as the attractiveness of a side-contract is affected by the existence of side-contracts between other agents: an agent may refuse to sign an otherwise beneficial side-contract if he suspects that a counterparty to the side-contract has an ulterior motive (e.g., a side-contract with a third party) that makes participation in the side-contract detrimental for the agent.

We relax the assumption mentioned above by developing a new, weaker, stability notion, which we shall refer to as *weak stability*. Weak stability incorporates considerations from the Nash equilibrium refinements literature into a concept of stability in the spirit of cooperative game theory. Under this notion, each side-contract can be viewed as if it were initiated by one of the deviating parties, say, agent i. An agent j who receives an offer to take part in this deviation conjectures what other deviation agent i may have initiated (with other agents) since it would have an effect on the attractiveness of i's offer. The only restriction we impose on agent j's conjecture is that it must *rationalise the observed offer*. We refer to such a conjecture as a *permissible* conjecture. Under weak stability, agent j rejects agent i's offer if there exists a permissible conjectured deviation that makes it detrimental for j to accept it.

Our main result is that, under mild domain restrictions, weakly stable collections of contracts are not constrained-efficient. We show that weakly stable collections of contracts exist, but that the contractual externalities constrain the agents' ability to transfer risk. It is worth pointing out that weak stability is defined by using conservative restrictions on the deviating agents' beliefs. If instead we were to use a stronger set of restrictions, then the amount of risk that could be transferred in a weakly stable collection of contracts would be even lower.

We present two applications in which we examine the implications of contractual externalities on the *volume* of trade. In both applications, we focus on collections that are robust to single and pairwise deviations (i.e., we restrict attention to bilateral side-contracts). In the first application, we study a reinsurance market in which external

[1] Similar assumptions are inherent in prominent stability notions that focus on one deviation at a time in the contexts of matching and network formation (e.g., Jackson and Wolinsky, 1996).

reinsurers provide coverage to primary insurers who are exposed to an aggregate shock.[2] We assume that only some of the primary insurers can manipulate the contractible variable and interpret the share of manipulators as a proxy for the level of corruption in the economy. For example, a high proportion of manipulators corresponds to an economy in which "revolving doors" between the public and private sectors are widespread. We derive a closed-form solution to the maximal level of risk-sharing that can be sustained by means of a weakly stable collection of contracts and show that it can be significantly lower than the constrained-efficient level of risk-sharing.

The second application studies speculative trade among risk-neutral speculators. Under an assumption that the economy is composed of two equally sized groups of optimistic and pessimistic agents, we derive a closed-form solution to the maximal volume of trade that can be sustained by means of a weakly stable collection of contracts and show that it is increasing when the agents' prior beliefs become more *polarised*. This is different from the case of bilateral speculative trade, in which the *magnitude* of the difference between the agents' beliefs has no effect on the volume of trade.

In both applications, the main message is that the maximal level of risk-sharing is U-shaped in the share of agents who can manipulate the contractible variable. That is, when corruption becomes more widespread in the economy, its effect on the maximal volume of trade is nonmonotone.

Related literature

This article is related to the risk-sharing networks literature. Bramoullé and Kranton (2007a,b) study network formation models in which agents mitigate risk by sharing their holdings with linked partners. In these models, the agents trade off between costly link formation and better risk-sharing. Bloch, Genicot, and Ray (2008) and Ambrus, Mobius, and Szeidl (2014) consider moral hazard in risk-sharing networks. In these models, ex post, agents who are supposed to make a transfer can deviate by refusing to do so. An agent who deviates loses some of his risk-sharing links. Bloch, Genicot, and Ray (2008) characterise stable risk-sharing networks while Ambrus, Mobius, and Szeidl (2014) study the extent and structure of risk-sharing.

Our work contributes to the literature on collusion. Laffont and Martimort (1997, 2000) develop a framework that incorporates collusion-proofness into mechanism design. In their models (as well as in Che and Kim, 2006), a fictitious third party

[2]Reinsurance instruments (e.g., catastrophe bonds) are often conditioned on state-dependent signals in order to avoid moral hazard in underwriting and claim settlements (see Doherty, 1997).

coordinates the side-contracts between the colluding agents. Earlier work on this topic focuses on the Vickrey–Clarke–Groves mechanism's vulnerability to collusion (Green and Laffont, 1979; Crémer, 1996). Bierbrauer and Hellwig (2016) show that coalition-proof mechanisms for public good provision that satisfy a robustness condition take the form of a voting mechanism. The implications of potential collusion have also been studied in the contexts of organisations (Tirole, 1986, 1992; Baliga and Sjöström, 1998; Mookherjee and Tsumagari, 2004; Celik, 2009) and auctions (Graham and Marshall, 1987; Jehiel and Caillaud, 1998; Marshall and Marx, 2007).

In this paper, contracts are contingent on a manipulable variable. Eliaz and Spiegler (2007, 2008, 2009) take a mechanism design approach to situations in which agents are motivated to bet on the state due to differences in their prior beliefs. In these models, the state is not verifiable and the agents can manipulate the contractible variable (a profile of actions) by incurring some cost. The ability to manipulate this variable creates incentive constraints that restrict the betting stakes. In Kahn and Mookherjee (1998), an insuree who is exposed to a private shock can purchase coverage from multiple insurers, where insurance contracts are negotiated sequentially according to an exogenously given protocol. Since there is no exclusive dealership and overinsurance may affect the insuree's incentives to exert effort (which in turn affects the contracts' outcomes), some insurers may be reluctant to provide the insuree with coverage.

Weak stability is related to farsighted-stability notions (see, e.g., Harsanyi, 1974; Chwe, 1994; Ray and Vohra, 2015) that characterise outcomes immune to deviations by players who recognise that their own deviations may trigger a chain of deviations by other players. In particular, in the context of network formation, pairwise farsighted-stability notions (e.g., Herings et al., 2009, 2019) have been used to extend Jackson and Wolinsky's (1996) notion of pairwise stability.[3]

Farsighted stability differs from weak stability in several aspects. First, under farsighted stability, deviations are deterred by potential *future* deviations. Second, under farsighted stability, the identity of the agent who initiates the deviation has no effect on the other agents' beliefs. Finally, pairwise farsighted-stability notions typically assume that deviations are observable to agents who are not part of the deviating coalition (e.g., they allow a deviation by a pair of agents (i, j) to trigger an additional deviation by a pair of agents $k \notin \{i, j\}$ and $l \notin \{i, j\}$).

The paper proceeds as follows. We present the model in Section 2 and analyse it

[3]Page et al. (2005) offer a different approach (that allows for deviations by coalitions of any size) to farsighted stability in networks.

in Section 3. In Section 4, we restrict attention to bilateral side-contracts. In Section 5, we study two applications of the model and Section 6 concludes. All proofs are relegated to Appendix A.

2 The Model

There is a set of agents $I = \{1, ..., n\}$, $n > 2$, and a set of states $\Theta = \{L, H\}$. Each agent $i \in I$ assigns probability π_i to the event that state H will be realised. We denote i's wealth in state θ by $w_i(\theta)$. Let $S = \{l, h\}$ be a set of signals that perfectly reveal the state unless there is some manipulation (the term "manipulation" will be clarified soon). We use $s(\theta)$ to denote the signal that results in state θ when there is no manipulation and, without loss of generality, assume that $s(H) = h$ and $s(L) = l$. Each agent i's preferences are represented by a concave and increasing vNM function $u_i : \mathbb{R} \to \mathbb{R}$.

State-contingent contracts are not feasible. Instead, agents can write signal-contingent contracts. A multilateral contract $g^K : S \to \mathbb{R}^{|K|-1}$ sets budget-balanced transfers among the members of $K \subseteq I$ contingent on the signal. We denote by $g_i^K(s)$ the transfer to agent $i \in K$ when the signal is s. A bilateral contract $b_{ij} : S \to \mathbb{R}$ sets a transfer $b_{ij}(s)$ from agent j to agent i contingent on the signal. Note that $b_{ij}(s) = g_i^{\{i,j\}}(s)$. We use B to denote the collection of contracts signed by the agents and $\mathcal{B}$ to denote the set of such collections. The collection $B + g^K$ is obtained by adding the contract g^K to B and the collection $B - g^K$ is obtained by dropping $g^K \in B$ from B. For every $B \in \mathcal{B}$, we use G^B to denote the multilateral contract that sums the transfers in B.

The timeline in the model is as follows. First, agents write signal-contingent contracts. After the contracting stage, a state is realised and the agents observe it. Subsequently, there is a manipulation stage in which some of the agents can try to affect the signal's realisation. Finally, agents receive transfers according to the contracts that they have signed and the signal that results from the manipulation stage.

The manipulation stage. After a state θ is realised and observed, each agent $i \in M \subseteq I$ can unilaterally change the signal from $s(\theta)$ to $s' \neq s(\theta)$ by paying a cost of $c > 0$. Let $\sigma : \mathcal{B} \times \Theta \to S$ be a manipulation function such that $\sigma(B, \theta)$ is the realisation of the signal at the end of the manipulation stage given a collection B and a state θ. For every collection of contracts B, let $PM(B) = \{m \in M | c < |G_m^B(h) - G_m^B(l)|\}$ be the set of potential manipulators. We say that B is *incentive compatible* (IC) if $PM(B) = \emptyset$. In such cases, there is no manipulation and $\sigma(B, \theta) = s(\theta)$ in every state

$\theta \in \Theta$.

If $|PM(B)| = 1$, then there is only one agent $i \in M$ with an incentive to manipulate the signal. If $s(\theta)$ does not match i's preferred realisation, then he will manipulate the signal and incur a cost of c. When $|PM(B)| > 1$, multiple agents have an incentive to manipulate the signal or to prevent others from doing so, as different agents may prefer different realisations. We now make two substantive assumptions about the signal that results from the manipulation stage and the cost of determining it when $|PM(B)| > 1$. These assumptions are inspired by the idea of truthful equilibria in menu auctions, which was developed by Bernheim and Whinston (1986) and later applied in the context of lobbying and economic influence (see, e.g., Grossman and Helpman, 1994; Dixit et al., 1997).

The first assumption is that if *all* members of $PM(B)$ prefer one realisation to the other, then they manipulate the signal to that realisation when necessary (e.g., when the state is H and their preferred realisation is l) and one of them incurs a cost of c. If there is no manipulation (i.e., when $\sigma(B, \theta) = s(\theta)$), no cost is incurred.

Assumption 1 *For every signal $s' \in S$ and every collection $B \in \mathcal{B}$ such that $PM(B) \neq \emptyset$ and $G_m^B(s') > G_m^B(s'')$ for every agent $m \in PM(B)$:*

- *$\sigma(B, \theta) = s'$ for every $\theta \in \Theta$.*
- *If $\sigma(B, \theta) \neq s(\theta)$, then there is an agent $i \in PM(B)$ who incurs a cost of c in addition to the transfer $G_i^B(\sigma(B, \theta))$ he obtains.*

The second assumption pertains to cases where there is a conflict of interest among the potential manipulators: some of them prefer the signal s' and the others prefer $s'' \neq s'$. We assume that if the potential manipulators who prefer realisation s'' have greater exposure to the signal than the potential manipulators who prefer realisation s', then the former impose their preferred realisation and incur a cost equal to the total exposure of the potential manipulators who prefer the other realisation. We assume that ties are broken in favour of h. For completeness, we assume that each potential manipulator pays a cost proportional to his exposure to the signal (our results are not sensitive to the particular sharing rule).

Assumption 2 *For every signal s', state θ, and collection of contracts $B \in \mathbb{B}$ such that there are two agents $i, j \in PM(B)$ for whom $G_i^B(s'') > G_i^B(s')$ and $G_j^B(s'') < G_j^B(s')$:*

- $\sigma(B,\theta) = h$ *if and only if* $\sum_{i \in PM(B)} (G_i^B(h) - G_i^B(l)) \geq 0$.
- *If* $\sigma(B,\theta) = s''$, *then each agent* $i \in \{i' \in PM(B) | G_{i'}^B(s'') > G_{i'}^B(s')\}$ *incurs a cost of*

$$\frac{\sum_{j \in \{i' \in PM(B) | G_{i'}^B(s') > G_{i'}^B(s'')\}} (G_j^B(s') - G_j^B(s''))(G_i^B(s'') - G_i^B(s'))}{\sum_{j \in \{i' \in PM(B) | G_{i'}^B(s') < G_{i'}^B(s'')\}} (G_j^B(s'') - G_j^B(s'))}$$

  in addition to the transfer $G_i^B(s'')$ *he obtains.*

Our approach in the present paper is to make elementary assumptions on how the signal is set and who incurs the cost of setting it. Alternatively, one can model the interaction at the manipulation stage as a noncooperative game. That is, one can commit to a specific game form and analyse its equilibria. Note that for a given game form, different collections of contracts may induce very different payoff functions as in other models of pregame contracting (see, e.g., Jackson and Wilkie, 2005). Further, since the set of manipulators can be a proper subset of the set of agents and we impose no constraint on the risk-sharing agreements, there are virtually no restrictions on the different payoff functions that can be induced by different collections of contracts.

For each $i \in I$, we use $\succ_i$ to denote i's *indirect preferences* over collections of contracts. The indirect preferences take ex-post manipulation into account. For example, suppose that B is IC and that $PM(B') = \{j\}$ and $G_j^{B'}(h) - G_j^{B'}(l) > c$. For $i \in I - \{j\}$, $B' \succ_i B$ if and only if

$$\begin{aligned} & \pi_i u_i \left(w_i(H) + G_i^B(h)\right) + (1-\pi_i) u_i \left(w_i(L) + G_i^B(l)\right) \\ < \; & \pi_i u_i \left(w_i(H) + G_i^{B'}(h)\right) + (1-\pi_i) u_i \left(w_i(L) + G_i^{B'}(h)\right). \end{aligned}$$

Note that in the expression on the RHS, agent i obtains $G_i^{B'}(h)$ in both states since $\sigma(B', L) = h$. For $i = j$, $B' \succ_i B$ if and only if

$$\begin{aligned} & \pi_i u_i \left(w_i(H) + G_i^B(h)\right) + (1-\pi_i) u_i \left(w_i(L) + G_i^B(l)\right) \\ < \; & \pi_i u_i \left(w_i(H) + G_i^{B'}(h)\right) + (1-\pi_i) u_i \left(w_i(L) + G_i^{B'}(h) - c\right). \end{aligned}$$

Observe that the manipulation cost c is taken into account only in state L, when j manipulates the signal. A collection of contracts B is said to be *individually rational* (IR) if each agent $i \in I$ prefers signing all of his contracts to not signing any contract. Following is the notion of efficiency that we use throughout the paper.

Definition 1 *A collection of contracts B is said to be constrained-efficient if it is IR, IC, and is not Pareto-dominated by another IC collection of contracts.*

3 Analysis

In this section, we present a notion of robustness against side-contracts, which we shall refer to as *multilateral stability.* Then, we show that IC multilaterally stable collections of contracts do not exist in two settings. This leads us to define a weaker notion of stability, which we shall refer to as *weak stability.* Finally, we show that weakly stable collections of contracts exist and that they are not constrained-efficient.

3.1 Multilateral Stability

Since signing a contract requires mutual consent, we study collections of contracts that are robust to both unilateral and multilateral deviations. We adopt a cooperative approach since it allows us to refrain from making assumptions about the process whereby contracts are negotiated. The following notion of stability is inspired by the network formation literature (see Jackson and Wolinsky, 1996).[4]

Definition 2 *A collection of contracts B is said to be multilaterally stable if the following two conditions are met:*

- *There exists no contract $g^K \in B$ and agent $i \in K$ such that $B - g^K \succ_i B$.*
- *There exists no contract $g^K \notin B$ such that $B + g^K \succ_i B$ for every $i \in K$.*

Multilateral stability requires that there be no agent who benefits from cancelling one of the contracts that he has signed, and that there be no group of agents who benefit from signing a new contract between them. When a group of agents $K \subseteq I$ writes a contract g^K and adds it to B, we refer to g^K as a *side-contract.* A group of agents can write a side-contract in order to provide each other with additional insurance or to *incentivise* one of them to manipulate the signal ex post.

[4]There are two differences between the present stability notion and Jackson and Wolinsky's notion. First, Jackson and Wolinsky's notion refers to single and bilateral deviations whereas the present notion allows also for multilateral deviations. Second, Jackson and Wolinsky's pairwise stability refers to binary links whereas, in the present paper, contracts are budget-balanced transfers.

Discussion: Side-contracts

We wish to stress that side-contracts are signal-contingent transfers just like any other contract. In particular, when a group of agents K write a side-contract g^K with the intention that $j \in K$ will manipulate the signal ex post, j's manipulation is not part of the side-contract officially (as such a clause would be difficult to enforce in the settings considered in this paper). Thus, for j to manipulate the signal ex post, the side-contract g^K must create the right incentives for it to be in j's interest to do so.

To see how two agents can use a side-contract to incentivise one of them to manipulate the signal without the manipulation officially being part of the contract, consider Example 1 and suppose that Alice and Bob receive a coverage of 90 dollars each, that is, $G^B_{Alice}(l) - G^B_{Alice}(h) = 90 = G^B_{Bob}(l) - G^B_{Bob}(h)$. As we showed in the Introduction, they can benefit from writing a side-contract in which Bob pays Alice a small $\epsilon > 0$ if and only if the appraiser's appraisal is l. This side-contract incentivises Alice to bribe the appraiser in state H (as the cost of doing so is 90 while the benefit is $G^B_{Alice}(l) - G^B_{Alice}(h) + \epsilon = 90 + \epsilon$), thereby guaranteeing Bob's preferred appraisal l. Observe that the side-contract between Alice and Bob is contingent on the appraisal rather than on whether Alice bribes the appraiser or not. Thus, if Alice's original coverage were 80 instead of 90, then she would not bribe the appraiser in state H as the benefit from doing so would be only $80 + \epsilon < 90$.

We now study two settings: speculative trade among risk-neutral agents and risk-sharing among risk-averse agents. In the first setting, agents trade to *increase* their exposure to the state because of the difference in their prior beliefs. In the second setting, agents trade to *reduce* their exposure to the state because of their risk aversion. In both of these settings there exists no collection of contracts that is both IC and multilaterally stable. These results demonstrate the instability inherent in settings where individuals write contracts contingent on a manipulable variable.

Proposition 1 *Suppose that $n > 3$, that all agents are risk-neutral, and that for any pair of agents i and j it holds that $\pi_i \neq \pi_j$. Then, there exists no collection of contracts that is both IC and multilaterally stable.*

Proposition 1 establishes that trade motivated purely by different prior beliefs cannot result in a collection of contracts that is both IC and multilaterally stable. The proof shows that if B is not constrained-efficient, then there are two agents who are better off writing a side-contract that increases their exposure to the signal. If B is constrained-efficient, then the agents' exposure to the signal must be high such that there is a pair of agents who are better off writing a side-contract that incentivises one of them to manipulate the signal ex post. That is, there are two agents who find it beneficial to collude to manipulate the signal ex post.

The next proposition considers a risk-sharing economy in which the agents' primary goal is to reduce their exposure to the state. We impose two mild domain restrictions. We refer to the first restriction as *richness*. In our model, there are four possible "types" of agents: manipulators or nonmanipulators with positive or negative initial exposure to the state, where agent i's *initial exposure* is $w_i(H) - w_i(L)$. A rich economy contains at least one agent of each type.

Definition 3 *The economy is said to satisfy richness if there are two agents $m, m' \in M$ such that $w_m(H) - w_m(L) > 0 > w_{m'}(H) - w_{m'}(L)$ and two agents $i, i' \notin M$ such that $w_i(H) - w_i(L) > 0 > w_{i'}(H) - w_{i'}(L)$.*

Note that richness rules out purely aggregate shocks (see, e.g., Example 1). We shall relax richness and study aggregate shocks in Section 5.

We refer to the second restriction as *nontriviality*. Essentially, nontriviality is an assumption that the manipulation cost is lower than the initial exposure (in absolute value) of at least two manipulators. Since the agents' primary goal here is to reduce their exposure to the state, if the manipulation cost is very high with respect to the agents' initial exposure to the state, manipulation becomes irrelevant and the model collapses to a conventional risk-sharing economy.

Definition 4 *The economy is said to satisfy nontriviality if there exist two agents $m, m' \in M$ such that $w_m(H) - w_m(L) \geq c$ and $w_{m'}(L) - w_{m'}(H) \geq c$.*

Proposition 2 *For each $i \in I$, let $\pi_i = \pi \in (0, 1)$ and let u_i be strictly concave. If nontriviality and richness are satisfied, then there exists no collection of contracts that is both IC and multilaterally stable.*

The proof of Proposition 2 shows that if a collection B provides inefficient insurance, then there is always at least one pair of agents who are better off coinsuring. The proof also shows that if B is constrained-efficient, then the agents' exposure to the signal is high such that there is at least one pair of agents who are better off writing a side-contract that incentivises one of them to manipulate the signal ex post; i.e., there are two agents who benefit from colluding to manipulate the signal ex post.

Note that in both of the above settings, given an IC collection of contracts, there is at least one pair of agents who benefit from writing a bilateral side-contract. This implies that even if we were to use a *pairwise stability* notion instead of a multilateral stability one, the impossibility results of Propositions 1 and 2 would hold.

The impossibility results follow from the fact that multilateral stability implicitly assumes that whenever a side-contract is beneficial for a group of agents, they will sign it. This assumption can be problematic when there are contractual externalities as the attractiveness of a side-contract is affected by the existence of side-contracts between other agents. For example, an agent j may refuse to sign an otherwise beneficial side-contract if he suspects that a counterparty to the deviation, who has already shown a tendency to steer away from the norm, has an ulterior motive that makes participating in the original deviation detrimental for j.

3.2 Weak Stability

We now develop a weaker stability notion that relaxes the above implicit assumption and takes into account suspicion of agents who initiate deviations from the original set of contracts. The idea underlying weak stability is that for a side-contract g^K to be signed it must be initiated by one of the contracting parties. An agent $j \in K$ who is approached by the initiator will refuse to sign g^K if he suspects that the initiator initiated an additional deviation (with agents other than j) that makes signing g^K detrimental for j. A side-contract g^K will not violate the weak stability of B if for any agent $i \in K$ who may initiate g^K there is an agent $j \in K - \{i\}$ who will suspect i and, therefore, refuse to participate in the side-contract.

Before we formally define weak stability and explain what makes an agent suspect the initiator of a deviation, we present an example in which healthy suspicion of the initiator's motivation is relevant.

Example 2. Let $I = \{1, ..., 8\}$, $\pi_1 > ... > \pi_8$, $u_i(z) = z$ for each $i \in I$, and $M = \{1, 2, 3, 6, 7, 8\}$. The table summarises the agents' transfers in a collection B.

Agent	1	2	3	4	5	6	7	8
$G_i^B(h) - G_i^B(l)$	c	c	c	0	0	$-c$	$-c$	$-c$

We present two side-contracts that violate the multilateral stability of B. The first side-contract is a bet between agents 4 and 5. We show that when this bet is initiated by agent 4, agent 5 has reason to suspect that agent 4 signed an additional side-contract with agent 3, thereby incentivising agent 3 to manipulate the signal ex post.

Suppose that agent 4 initiates a side-contract b_{45} such that $b_{45}(h) > 0 > b_{45}(l)$. Agent 5 might suspect that agent 4 has initiated another side-contract b_{34} such that $b_{34}(h) = \varepsilon > 0 = b_{34}(l)$, that is, a deviation in which agent 4 incentivises agent 3 to manipulate the signal from l to h by paying him $\epsilon > 0$ if and only if the realised signal is h. Note that $B + b_{34} + b_{45} \succ_4 B + b_{34}$ (i.e., given b_{34}, adding the contract b_{45} makes agent 4 better off). Agreeing to b_{45} exposes agent 5 to a negative externality imposed by agent 3's manipulation of the signal as a result of b_{34}. Thus, given b_{34}, agent 5 is worse off agreeing to b_{45}.

We now present a second deviation in which agents 6 and 7 write a side-contract with the intention that agent 7 will manipulate the signal from h to l in state H. When agent 6 initiates this side-contract, agent 7 may suspect that agent 6 has an ulterior motive in the form of an additional side-contract with agent 8 that incentivises agent 6 to manipulate the signal himself. That is, agent 7 may suspect that agent 6 is using their side-contract b_{76} to make agent 7 manipulate the signal and pay the manipulation cost instead of doing so himself. In other words, agent 7 may suspect that agent 6 is trying to free-ride on him.

Suppose that agent 6 initiates a side-contract b_{76} such that $b_{76}(l) = \epsilon > 0 = b_{76}(h)$. This side-contract incentivises agent 7 to manipulate the signal to l in state H. Agent 7 may suspect that agent 6 has also initiated a side-contract b_{68} such that $b_{68}(s) = b_{76}(s)$ for each $s \in S$. Observe that $G_6^B(l) - G_6^B(h) + b_{68}(l) - b_{68}(h) = c + \epsilon > c$. Assumption 1 implies that $\sigma(B + b_{68}, \theta) = \sigma(B + b_{68} + b_{76}, \theta) = l$ for each $\theta \in \Theta$. While the realised signal is the same whether or not agent 7 agrees to b_{76}, agreeing to b_{76} makes agent 7 pay the cost of manipulation instead of agent 6 paying it. If ϵ is small relative to c, agent 7 is worse off agreeing to agent 6's offer to deviate. Moreover, when ϵ is small relative to c, $B + b_{68} + b_{76} \succ_6 B + b_{68}$ since agent 6 does not pay for manipulation under $B + b_{68} + b_{76}$. Thus, the side-contract b_{76} that agent 7 observes can be rationalised by a conjecture that 6 signed an additional side-contract b_{68}.

We now develop a notion of stability that takes into account the suspicion motive presented above. Given a collection B, a side-contract g^K, and an initiator $i \in K$, agent $j \in K - \{i\}$ can form a conjecture about an additional deviation by agent i. Specifically, agent j's conjecture can be either that agent i signed a new side-contract $g^{K'}$ such that $i \in K'$ and $j \notin K'$, or that he unilaterally cancelled an existing contract $g^{K'} \in B$ such that $i \in K'$ and[5] $j \notin K'$. We denote agent j's conjecture by $\beta_j(B, g^K, i)$. In the former case, $\beta_j(B, g^K, i) = g^{K'}$ and in the latter case $\beta_j(B, g^K, i) = -g^{K'}$. We denote by $\beta_j^{-1}(B, g^K, i)$ agent i's counterparties to the conjectured deviation $\beta_j(B, g^K, i)$. Since we keep the collection of contracts fixed throughout the analysis, we shall omit the collection of contracts from the description of a conjecture and write $\beta_j(g^K, i)$ instead of $\beta_j(B, g^K, i)$.

We say that the conjecture $\beta_j(g^K, i)$ *blocks* the side-contract g^K if $B + \beta_j(g^K, i) \succ_j B + \beta_j(g^K, i) + g^K$. The notion of blocking applies to conjectured side-contracts as well. That is, the conjecture $\beta_k(g^{K'}, i)$ blocks the conjecture $\beta_j(g^K, i)$ if $\beta_j(g^K, i) = g^{K'}$, $k \in K' - \{i\}$, and $B + \beta_k(g^{K'}, i) \succ_k B + \beta_k(g^{K'}, i) + g^{K'}$. Note that a cancellation of a contract cannot be blocked as it does not require mutual consent.

We shall refine the set of conjectures agent j can form by introducing a mild consistency requirement. Agent j's conjecture $\beta_j(g^K, i)$ is consistent if the addition of g^K to $B + \beta_j(g^K, i)$ makes the initiator, agent i, better off. In other words, agent j's conjecture is consistent with the offer he observes if it can rationalise it. As an illustration, consider the first deviation presented in Example 2, b_{45}. As we showed in the example, $B + b_{34} + b_{45} \succ_4 B + b_{34}$ and, therefore, the conjecture $\beta_5(b_{45}, 4) = b_{34}$ is consistent with an offer to sign the side-contract b_{45} made by agent 4.

Definition 5 *A conjecture $\beta_j(g^K, i)$ is consistent if $B + \beta_j(g^K, i) + g^K \succ_i B + \beta_j(g^K, i)$.*

Consistency assumes that an agent who receives an offer to deviate takes into account the proposer's motivation. However, the receiving agent might want to take into account other agents' motivations as well. For example, imagine that agent i offers agent j the opportunity to sign a bilateral side-contract b_{ij} and consider a bilateral side-contract b_{ik} such that the conjecture $\beta_j(b_{ij}, i) = b_{ik}$ is consistent. Should agent j form such a conjecture? Since side-contracts require mutual consent, the answer depends on whether agent j thinks that agent k is willing to sign b_{ik}. Let us assess k's willingness to sign b_{ik}. If k can come up with a consistent conjecture $\beta_k(b_{ik}, i)$ that blocks b_{ik}, then k might refuse to sign b_{ik}. Again, since side-contracts require mutual consent, k's decision may depend on whether he thinks that i's counterparties to his conjectured deviation, $\beta_k^{-1}(b_{ik}, i)$, are willing to participate in it. This process can be iterated over and over again. Thus, assessing whether k is willing to sign b_{ik} requires j to take the motivations of all the other agents into account.

[5] In addition, agent j is also allowed to conjecture that agent i is not engaged in an additional deviation. We use $\beta_j(B, g^K, i) = \emptyset$ to denote such a conjecture.

We now complete the presentation of our stability notion by describing how each agent takes all the other agents' motivations into account. Under our notion, an agent rejects an offer to sign a side-contract if (i) there is a consistent conjecture that blocks it, and (ii) this consistent conjectured deviation is not blocked by any other consistent conjecture, which itself is not blocked by any other consistent conjecture, and so on ad infinitum. We refer to consistent conjectures that satisfy (ii) as *permissible*.

Definition 6 *For every collection $B \in \mathcal{B}$, side-contract g^K, and agents $i, j \in K$, let $A_j^0(B, g^K, i)$ be the set of consistent conjectures $\beta_j(g^K, i)$ that i cancelled a contract $g^{K'} \in B$ such that $i \in K'$ and $j \notin K'$. For every $t > 0$, let $A_j^t(B, g^K, i)$ be the set of consistent conjectures $\beta_j(g^K, i)$ that satisfy the following condition:*

- *If there exists an agent $k \in \beta_j^{-1}(g^K, i)$ and a consistent conjecture $\beta_k(\beta_j(g^K, i), i)$ that blocks $\beta_j(g^K, i)$, then there is some agent $z \in \beta_k^{-1}(\beta_j(g^K, i), i)$ and a conjecture $\beta_z(\beta_k(\beta_j(g^K, i), i), i) \in \cup_{x=0}^{t-1} A_z^x(B, \beta_k(\beta_j(g^K, i), i), i)$ that blocks $\beta_k(\beta_j(g^K, i), i)$.*

A conjecture $\beta_j(g^K, i)$ is said to be permissible if $\beta_j(g^K, i) \in \cup_{t=0}^{\infty} A_j^t(B, g^K, i)$.

Since the conjectures in $A_j^0(B, g^K, i)$ cannot be blocked, consistency is sufficient for permissibility in this case. This enables us to construct the set of permissible conjectures in a unique manner. The construction of the set of permissible conjectures guarantees that a permissible conjecture can never be blocked by another permissible conjecture. This property corresponds to internal consistency in von Neumann-Morgenstern stable sets (see von Neumann and Morgenstern, 1944).

Definition 7 *A collection of contracts B is said to be weakly stable if the following two conditions are met:*

- *There exists no contract $g^K \in B$ and agent $i \in K$ such that $B - g^K \succ_i B$.*
- *For every side-contract $g^K \notin B$ such that $B + g^K \succ_i B$ for some $i \in K$, there exists an agent $j \in K - \{i\}$ and a permissible conjecture $\beta_j(g^K, i)$ that blocks g^K.*

Note that weak stability requires that every side-contract be blocked for any agent who initiates it. For instance, if $B + b_{ij} \succ_i B$ and $B + b_{ij} \succ_j B$, then the weak stability of B requires that the side-contract b_{ij} be blocked both by a permissible conjecture $\beta_j(b_{ij}, i)$ and by a permissible conjecture $\beta_i(b_{ij}, j)$. Thus, the same side-contract can be treated differently when the identity of its initiator is different.

Discussion: Solution concept

Alternative stability notions. Let us consider the possibility of using a few "off the shelf" notions of stability. For example, when considering multilateral deviations, it is natural to think of cooperative notions such as the core or the Aumann–Maschler bargaining set (Aumann and Maschler, 1964). However, such notions cannot be used to examine the effect of *adding new contracts to an existing set* of contracts as the idea underlying such notions is that coalitions deviate to a state of autarchy. This is also the idea underlying the self-enforcing risk-sharing agreements in Genicot and Ray (2003). Concepts such as strong Nash equilibrium (Aumann, 1959), coalition-proof Nash equilibrium (Bernheim et al., 1987), and coalitional rationalisability (Ambrus, 2006) can capture the idea that a coalition of agents deviates whereas agents who are not members of the deviating coalition do not change their behaviour. However, since these concepts are noncooperative, they require strong assumptions about the way contracts and deviations are negotiated. Moreover, the existence of a strong Nash equilibrium or a coalition-proof Nash equilibrium is not guaranteed.

Robustness. Our stability notion assumes that agents form "pure" conjectures. In the Supplementary Appendix, we show that the results do not depend on this assumption. In fact, we show that a collection of contracts is weakly stable when mixed conjectures are allowed, if and only if it is weakly stable when mixed conjectures are not allowed.

Our stability notion also assumes that agents form conjectures of a single deviation. We can extend this notion by allowing agents to form conjectures of multiple deviations. For example, an agent who is approached by another agent i could entertain a belief that agent i initiated two additional side-contracts, each with a different group of agents. While this extension is beyond the scope of this paper, it is possible to show that it would not change any of our results.

Relation to the Nash equilibrium refinements literature. Underlying the notion of consistency is a *forward-induction* logic in the spirit of a strand of the Nash equilibrium refinements literature and, in particular, the intuitive criterion (Cho and Kreps, 1987).[6] The intuitive criterion is built on a forward-induction argument that restricts the beliefs of an agent who receives a message off the equilibrium path. Specifically, the receiver's belief can assign positive probability only to types of senders who can obtain a payoff strictly higher than their equilibrium payoff by sending the off-path message under the

[6] Pomatto (2019) applies similar arguments to test the stability of two-sided matchings.

assumption that the receiver best responds to any belief.

The consistency requirement applies a similar logic to refine the beliefs of an agent who receives an offer to deviate from the existing collection of contracts. To see the analogy, note that the existing collection can be interpreted as an equilibrium, an offer to sign a side-contract can be interpreted as an off-path message by the proposer (initiator), the additional deviation by the proposer can be viewed as his "type," and the offer's receiver's conjecture can be interpreted as his belief about the proposer's type. Essentially, consistency restricts the receiver to pure conjectures that assign positive probability only to types of initiators who can strictly benefit from offering the side-contract.

3.2.1 Main Results

In this subsection, we provide the main results of the paper. First, Proposition 3 shows that weakly stable collections exist. Second, Proposition 4 establishes that under mild domain restrictions weakly stable collections are not constrained-efficient.

Proposition 3 *If $|M| > 2$, then there exists an IR, IC, and weakly stable collection of contracts.*

In the proof, we construct a collection of contracts B and show that it is weakly stable. Each agent's transfers in B sum to zero, which makes it IC and IR by definition.[7] Note, however, that the collection constructed in the proof is not the null contract. In fact, the agents' ability to unilaterally cancel previously signed contracts plays an important role in this proof: agents reject otherwise beneficial offers to sign side-contracts based on a suspicion that their counterparties to the side-contracts cancelled a previously signed contract.

The collection B is constructed such that for any side-contract g^K that makes an agent $i \in K$ better off, one of i's counterparties to the deviation, agent k, has a permissible conjecture that i cancelled a previously signed contract $g \in B$ that blocks g^K. Cancelling g induces manipulation (not necessarily by i) that makes it detrimental for k to sign g^K. The main challenge in the construction of B is to balance between the requirement that k's conjecture be permissible and the requirement that $B - g \not\succ_i B$, which is necessary to satisfy the first requirement of weak stability.

[7] If we were to consider a collection of contracts in which agents' transfers do not sum to zero, then it would be impossible to determine whether or not the collection is IR without making additional assumptions about the agents' preferences, beliefs, and endowments.

The next proposition emphasises the tension between stability and efficiency in the context of a conventional risk-sharing economy. It shows that despite the weak requirements of the solution concept, under fairly general conditions, there exists no collection of contracts that is both weakly stable and constrained-efficient.

Proposition 4 *Let richness and nontriviality hold and suppose that* $\pi_i = \pi \in (0, 1)$ *and* u_i *is strictly concave for each* $i \in I$. *If a collection of contracts* B *is constrained-efficient, then it is not weakly stable.*

The proof relies on Lemma 2, which describes a side-contract that cannot be blocked by any consistent conjecture. In this side-contract, an agent who can manipulate the signal *colludes* with an agent who cannot do so to set the signal to their preferred realisation ex post. In the collusive side-contract, the agent who cannot manipulate the signal makes positive *signal-contingent* payments to the manipulator that incentivise the latter to manipulate the signal ex post, if necessary. In the proof, we show that if B is constrained-efficient, then there exists such a collusive side-contract that makes both counterparties better off.

The fact that the above deviation involves agents with *heterogeneous strategic capabilities* plays a key role. In particular, it is important that the deviation is initiated by an agent who cannot manipulate the signal as it restricts the set of conjectures his counterparty to the deviation can hold. Intuitively, it is harder for the latter agent to suspect that the initiator has an ulterior motive when the initiator is not a manipulator.

As an illustration, suppose that agent i offers agent $m \in M$ the opportunity to sign a bilateral side-contract that incentivises m to manipulate the signal ex post. If $i \in M$, then agent m can form a conjecture that agent i is trying to free-ride on him, namely, trying to make agent m pay the manipulation cost instead of doing so himself (as in the second deviation in Example 2). If $i \notin M$, then agent m cannot form such a conjecture. Thus, agent i's inability to manipulate the signal restricts the set of conjectures agent m can form, which may enable the two agents to collude.

4 Pairwise Weak Stability

So far, we have placed no restriction on the size of the deviating coalition. In this section, we focus on single and pairwise deviations, in line with the network formation literature. Another motivation for the focus on single and pairwise deviations is that deviations that hinge on the cooperation of a large number of players may be more difficult to coordinate. We now examine whether the restriction to pairwise deviations can alleviate the tension between stability and efficiency that was established in the previous section.

First, we need to adapt our basic notions. Given a collection B and a bilateral side-contract b_{ij}, a conjectured deviation $\beta_j(b_{ij}, i)$ can take the form of either the signing of a new bilateral side-contract b_{ik} between agent i and an agent $k \in I - \{i, j\}$ or the unilateral cancellation of an existing contract $g^k \in B$ such that $i \in K$ and $j \notin K$. Since we focus on bilateral side-contracts, we can omit the initiator's label from the description of a conjecture and write $\beta_j(b_{ij})$ instead of $\beta_j(B, b_{ij}, i)$. The notions of consistency, blocking, and permissibility remain as in the previous section. Following is the adapted stability notion.

Definition 8 *A collection of contracts B is said to be pairwise weakly stable if the following two conditions are met:*

- *There is no agent $i \in I$ and contract $g^K \in B$ such that $i \in K$ and $B - g^K \succ_i B$.*
- *For every bilateral side-contract b_{ij} such that $B + b_{ij} \succ_i B$ there exists a permissible conjecture $\beta_j(b_{ij})$ that blocks it.*

Pairwise weak stability is not necessarily a coarsening of weak stability as these notions may induce different sets of permissible conjectures (specifically, a conjecture can be permissible when weak stability is used and not permissible when pairwise weak stability is used). Nonetheless, the proof of the existence result (Proposition 3) holds for pairwise weakly stable collections as well. The reason for this effect is that the conjectured deviations that stabilise the collection of contracts constructed in the proof are unilateral; i.e., agents reject offers to deviate because they suspect that the initiator of the deviation cancelled an existing contract.

Pairwise weak stability is not a refinement of weak stability, as the set of admissible deviations is smaller under pairwise weak stability. Nonetheless, the proof of the impossibility result (Proposition 4) holds for pairwise weakly stable collections as well. Essentially, the proof shows that for any constrained-efficient collection of contracts there exists a bilateral side-contract that destabilises it and cannot be blocked by any *consistent* conjecture. Since the set of consistent conjectures under weak stability is a superset of the set of consistent conjectures under pairwise weak stability, which itself is a superset of the set of permissible conjectures under pairwise weak stability, the proof of Proposition 4 holds. The next corollary summarises the short discussion above.

Corollary 1 *Suppose that richness and nontriviality hold. If* $|M| > 2$*, then there exists a collection of contracts that is IR, IC, and pairwise weakly stable. However, if* $\pi_i = \pi \in (0, 1)$ *and* u_i *is strictly concave for each* $i \in I$*, then every constrained-efficient collection of contracts is not pairwise weakly stable.*

5 Applications

We now present two applications of the model. In the first application we study risk-sharing in a reinsurance market and in the second application we study risk-bearing when agents are motivated by different prior beliefs. In both applications, the main messages are that the maximal level of risk that can be transferred by means of *pairwise weakly stable* collections of contracts is nonmonotone with respect to the share of manipulators and that it can be substantially lower than the constrained-efficient level of risk-sharing. The focus on pairwise deviations allows us to highlight the latter point by deriving a closed-form solution to the volume of trade.[8]

5.1 Reinsurance

We study a reinsurance market in which local insurers who are exposed to a shock receive coverage from external reinsurers who are not directly exposed to the shock. The external reinsurers can also be thought of as capital market investors. The signal can be interpreted as a local governor's declaration of a state of emergency and manipulation can be interpreted as lobbying. In practice, reinsurance contracts and instruments (e.g., catastrophe bonds) are typically contingent on state-dependent signals and not on actual losses incurred by insurers to prevent moral hazard problems in underwriting and claim settlements (see Doherty, 1997). We assume that some of the local insurers can manipulate the contractible variable while the external reinsurers cannot do so, and interpret the share of local insurers who have the ability to manipulate the contractible variable as a proxy for the level of corruption in the economy.

[8]It is possible to show that the amount of risk that can be transferred by means of weakly stable collections of contracts is lower than the amount of risk that can be transferred by means of pairwise weakly stable collections of contracts.

To model this reinsurance market, we partition the set of agents I into a set of local insurers L and a set of external reinsurers E and assume that $M \subseteq L$. To capture the idea that the local insurers are exposed to a high-volume catastrophe, we set $w_i(H) - w_i(L) = w \geq c$ for each $i \in L$. We shall assume that the cardinality of E is large relative to that of L such that the external reinsurers can absorb all the risk in the economy. Specifically, we assume that $\frac{w}{c} < \frac{|E|}{|L|}$.

To avoid frictions arising from the discreteness of L, we assume that there are many local insurers and denote the share of manipulators $\frac{|M|}{|L|}$ by α. For the sake of tractability, we assume that the local insurers exhibit constant absolute risk aversion (CARA). That is, for each $i \in L$, $u_i(z) = -exp(-\gamma z)$, $\gamma > 0$. To simplify the exposition, it is also assumed that each $i \in E$ is risk-neutral and that $w \leq c + \frac{1}{\gamma} log(\pi + (1-\pi) exp(\gamma w))$.

We start the analysis by showing that, unless all the local insurers can manipulate the signal or none can, pairwise weakly stable collections of contracts are not constrained-efficient. Then, we show that the maximal level of risk that can be shared by means of such collections is U-shaped in the share of local insurers who can manipulate the signal and increasing in the agents' risk aversion. In the Supplementary Appendix, we provide a closed-form solution to this maximal level, and show that it can be significantly lower than the level of coverage agents obtain in constrained-efficient collections of contracts.

The first result of this section is based on an argument similar to the one used in Proposition 4 and does not rely on the CARA assumption.

Proposition 5 *If $\alpha \in (0,1)$, then there exists no collection of contracts that is both constrained-efficient and pairwise weakly stable.*

Proposition 5 shows that, except for extreme values of α, pairwise weakly stable collections are not constrained-efficient. In the Supplementary Appendix, we show that for extreme values of α, the amount of risk that can be transferred by means of pairwise weakly stable contracts coincides with the constrained-efficient level of coverage.

We define insurer i's coverage as $min\left\{G_i^B(l) - G_i^B(h), w\right\}$. Note that our definition does not allow for over-insurance. As a result, the aggregate level of coverage in a collection B is $\sum_{i \in L} min\left\{G_i^B(l) - G_i^B(h), w\right\}$. The next result establishes that the maximal aggregate coverage the local insurers can obtain by means of a pairwise weakly stable, IC, and IR collection of contracts is U-shaped in α.

Proposition 6 *Let $|M| > 2$. There exists an $\alpha^* \in (0,1)$ such that the maximal aggregate coverage that can be obtained using an IR, IC, and pairwise weakly stable collection is increasing in α for $\alpha > \alpha^*$ and decreasing in α for $\alpha < \alpha^*$.*

The first part of the proof is essentially an application of Lemma 2.[9] It shows that in a pairwise weakly stable collection of contracts there cannot be a pair of local insurers, $m \in M$ and $i \in L - M$, such that i is willing to pay more than $c - (G_m^B(l) - G_m^B(h))$ to guarantee the realisation l ex post. If such a pair were to exist, i and m would benefit from writing a side-contract by which i makes signal-contingent payments to m that incentivise m to manipulate the signal ex post. The lemma shows that such a collusive side-contract would violate the pairwise weak stability of the collection. We translate this restriction to constraint (8).

In the second part of the proof, we solve for the maximal coverage that can be obtained subject to incentive compatibility and (8). We show that under the CARA assumption, the problem is equivalent to maximising a convex combination (with weights α and $1-\alpha$) of the coverage provided to manipulators and the coverage provided to nonmanipulators subject to a concave constraint, which yields the U-shaped structure. In the final part of the proof, we construct an IR collection that induces that maximal level of coverage and show that there are no additional deviations that violate the collection's pairwise weak stability.

Comparative statics: Risk aversion

To examine the effect of risk aversion on the level of coverage, consider the willingness of an agent $i \in L - M$ to pay to guarantee that the signal will be l, $z(G_i^B(l) - G_i^B(h))$, which is given in (6). The more risk averse i is, the less willing he is to pay for manipulation. To see why, observe that when agent $i \in L - M$ incentivises agent $m \in M$ to manipulate the signal by paying him x if $s = l$, it is as if agent i were giving up on a state-dependent coverage in return for a sure transfer of $G_i^B(l) - x$. When agent i becomes more averse to risk, the state-dependent coverage becomes more attractive than the sure transfer, such that i's willingness to pay for manipulation decreases. Thus, increasing the agents' level of risk aversion increases the amount of coverage nonmanipulators can obtain without violating constraint (8) and thus the collection's pairwise weak stability.

[9] Since the set of consistent conjectures under pairwise weak stability is a subset of the set of consistent conjectures under weak stability, the proof of Lemma 2 holds when agents are restricted to bilateral deviations.

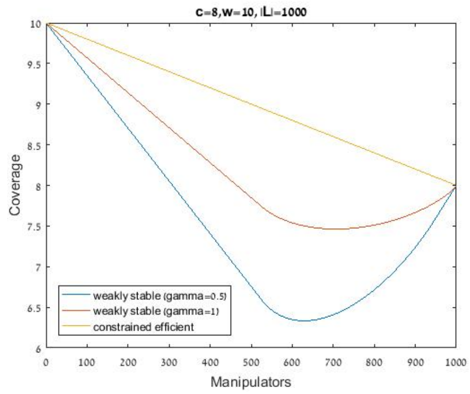

Figure 1: $c = 8$

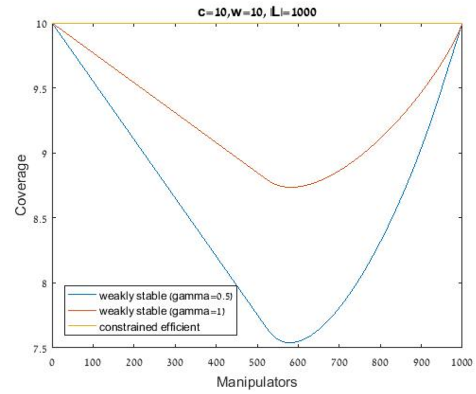

Figure 2: $c = 10$

In the Supplementary Appendix we illustrate the effects of α and γ on the aggregate coverage by solving the model analytically. The primary goal of the analysis in the Supplementary Appendix is to show that the adverse effect on the aggregate coverage is of a first-order magnitude. In Figure 1 we illustrate the maximal average level of coverage for $w = 10$, $c = 8$, $|L| = 1000$, $\gamma = 0.5$, and $\gamma = 1$ versus the constrained-efficient level of coverage. Figure 2 illustrates the results for $c = 10$, and demonstrates that even when $c = w$ such that in every constrained-efficient collection of contracts the agents are fully covered, the maximal level of coverage that can be obtained using a pairwise weakly stable collection can be significantly lower than the constrained-efficient level of coverage.

5.2 Speculative Trade

This subsection focuses on reallocation of risk motivated by different prior beliefs. The signal in this section can be interpreted as an event or as a financial benchmark that the agents are not directly exposed to. We explore the maximal volume of speculative trade that can be sustained by means of a pairwise weakly stable, IC, and IR collection of contracts, and show that it is nonmonotone in the share of agents who can manipulate the signal and is increasing when the agents' prior beliefs become more polarised.

To focus on speculation, we assume that all of the agents are risk-neutral. We impose additional structure on the problem by assuming that there are two types of agents. These types differ from each other in their prior beliefs, whereas the share of manipulators in each type of agent is identical. Formally, we partition I into two disjoint groups of equal size, I^h and I^l, and assume that each $i \in I^l$ has a prior belief

π_l and each $i \in I^h$ has a prior belief $\pi_h > \pi_l$. We assume that $|M \cap I^h| = |M \cap I^l|$. To avoid integer problems, we set $\alpha := \frac{|M|}{|I|}$.

The *volume of speculative trade* induced by a collection B is $\sum_{i \in I} |G_i^B(h) - G_i^B(l)|$. We now derive a closed-form solution to the maximal volume of speculative trade that can be obtained using IR, IC, and weakly stable contracts. To focus on speculation motivated by non-common priors (rather than on speculative trade between two members of I^h or speculative trade between two members of I^l), we restrict our attention to collections that satisfy the following condition.

Condition 1 *A collection of contracts B satisfies Condition 1 if $G_i^B(h) \geq G_i^B(l)$ for each $i \in I^h$ and $G_i^B(l) \geq G_i^B(h)$ for each $i \in I^l$.*

Proposition 7 *Let $\alpha \in (0,1)$ and $|M| > 2$. The maximal average volume of speculative trade that can be sustained by means of an IR, IC, and pairwise weakly stable collection of contracts that satisfies Condition 1 is*

$$max\left\{\alpha c, (1-\alpha) min\left\{\frac{c}{1-\pi_h}, \frac{c}{\pi_l}\right\}\right\}. \tag{1}$$

The maximal volume of speculative trade is U-shaped in the proportion of manipulators. It is weakly increasing in π_h and weakly decreasing in π_l. That is, when the agents' beliefs are more polarised, there is room for more speculative trade.

The intuition behind the nonmonotonicity of the volume of trade w.r.t. α is similar to the that behind the U-shaped level of coverage obtained in Proposition 6. There are two differences, however. First, the linearity of the utility functions implies that the constraint implied by Lemma 2 is linear rather than strictly concave. Thus, in a collection that maximises the volume of trade, either only nonmanipulators trade or only manipulators trade. This results in the maximum in expression (1). The second key difference is that there are no external agents who can absorb the positions of the members of I^h or I^l, which results in the minimum in expression (1).

The effect of polarisation: A comparison to bilateral trade

When $n = 2$ there are no contractual externalities and so the magnitude of the difference between the agents' beliefs has no effect on the volume of trade. If one of the agents can manipulate the signal, then the volume of trade is $2c$ as a higher volume would lead to manipulation. If none of the agents can manipulate the signal, then they will always want to scale up the stakes of the bets between them.

When $n > 2$, agents can collude to manipulate the signal ex post. When $G_i^B(h) - G_i^B(l) > 0$, agent $i \in I^h$ is willing to pay $(1 - \pi_h)(G_i^B(h) - G_i^B(h))$ to guarantee that the signal will be h even in state L. Intuitively, the more likely agent i thinks state H is, the less willing he is to pay for manipulation in state L. Thus, the higher π_h is, the greater the exposure $G_i^B(h) - G_i^B(l)$ agent i can have without destabilising the collection of contracts. Analogously for $i \in I^l$, the lower π_l is, the greater the exposure $G_i^B(l) - G_i^B(h)$ agent i can have. We can conclude that the more extreme the agents' priors are, the less willing the agents are to pay for manipulation, which, all else equal, enables them to hold a larger speculative position without violating the pairwise weak stability of the collection of contracts.

6 Concluding Remarks

The present paper studied reallocation of risk by means of contracts that are contingent on manipulable variables. This manipulability creates a moral hazard problem that limits the ability of agents who can manipulate the signal to share risk. The reason for this effect is that other individuals take into account the possibility of ex-post manipulation and, therefore, are less inclined to share risk with these agents.

The multilateral nature of our setting creates contractual externalities that magnify the moral hazard problem and limit all agents' ability to share risk, regardless of whether they can manipulate the signal. In the presence of these externalities, an agent who is considering whether to share risk with another individual has to take into account the possibility that this individual will collude with a third party to manipulate the signal ex post. Our analysis shows that the magnitude of the contractual externalities' negative effect on the overall level of risk-sharing can be substantial and may depend on the share of agents who can manipulate the signal nonmonotonically.

Our assumption about the richness of the economy ruled out cases in which only one agent is exposed to a shock and he is the only one who can manipulate the signal. Such cases are common in practice. For instance, standard insurance contracts are typically conditioned on an insuree's report about the occurrence of the shock. To incorporate such cases into our framework, we can think of c as the cost of reporting that a shock occurred when it did not occur and interpret the assumption that $n > 2$ as the agent's ability to purchase coverage from multiple insurers. It can be shown that if the insurers and the agent have the same prior beliefs, then multilaterally stable collections of contracts exist and they are constrained-efficient. The reason for this is that a collusive side-contract that incentivises the agent to manipulate the contractible variable ex post cannot make any insurer better off. Thus, such a side-contract cannot violate the multilateral stability of the collection of contracts. On the other hand, a side-contract in which the agent purchases additional coverage can violate the multilateral stability of a collection that is not constrained-efficient. Thus, we obtain constrained-efficient risk-sharing in this case.

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Appendix A: Proofs

The proofs of Propositions 1 and 2 rely on the following lemma.

Lemma 1 *Let B be an IC collection of contracts. If there exists a pair of agents, $m \in M$ and $i \neq m$, such that $|G_m^B(h) - G_m^B(l)| = c$ and sign $\left(G_m^B(h) - G_m^B(l)\right)$ = sign $\left(G_i^B(h) - G_i^B(l)\right)$, then B is not multilaterally stable.*

Proof. Without loss of generality, assume that $G_m^B(h) - G_m^B(l) = c$ and $G_i^B(h) > G_i^B(l)$. Consider a side-contract $\hat{b}_{mi}$ such that $c \geq \hat{b}_{mi}(h) > \hat{b}_{mi}(l) = 0$. Observe that $G_m^B(h) - G_m^B(l) + \hat{b}_{mi}(h) > c$. If $i \in M$, then it must be that $c > G_i^B(h) - G_i^B(l) - \hat{b}_{mi}(h) > -c$ since B is IC. It follows that $PM\left(B + \hat{b}_{mi}\right) = \{m\}$. By Assumption 1, $\sigma(B + \hat{b}_{mi}, \theta) = h$ for every $\theta \in \Theta$. If $\hat{b}_{mi}(h)$ is sufficiently close to 0, then

$$\pi_i u_i(w_i(H) + G_i^B(h) - \hat{b}_{mi}(h)) + (1 - \pi_i) u_i(w_i(L) + G_i^B(h) - \hat{b}_{mi}(h)) > \\ \pi_i u_i(w_i(H) + G_i^B(h)) + (1 - \pi_i) u_i(w_i(L) + G_i^B(l))$$

since $G_i^B(h) > G_i^B(l)$. Agent m is also better off signing the side-contract $\hat{b}_{mi}$ since

$$\pi_m u_m(w_m(H) + G_m^B(h) + \hat{b}_{mi}(h)) + (1 - \pi_m) u_m(w_m(L) + G_m^B(h) - c + \hat{b}_{mi}(h)) = \\ \pi_m u_m(w_m(H) + G_m^B(h) + \hat{b}_{mi}(h)) + (1 - \pi_m) u_m(w_m(L) + G_m^B(l) + \hat{b}_{mi}(h)) > \\ \pi_m u_m(w_m(H) + G_m^B(h)) + (1 - \pi_m) u_m(w_m(L) + G_m^B(l)).$$

Hence, B is not multilaterally stable. ∎

Proof of Proposition 1

First, we consider the case where $|I - M| \leq 1$. Assume to the contrary that B is IC and multilaterally stable, and consider three agents $k, m, i \in M$ such that $\pi_k > \pi_i > \pi_m$.

Assume to the contrary that both $G_m^B(l) - G_m^B(h) < c$ and $G_i^B(h) - G_i^B(l) < c$. To obtain a contradiction to the multilateral stability of B, we construct a side-contract b_{mi} such that $B + b_{mi} \succ_i B$ and $B + b_{mi} \succ_m B$. Let $b_{mi}(l) = \epsilon$ and $b_{mi}(h) = \frac{\hat{\pi}-1}{\hat{\pi}}\epsilon$, where $\hat{\pi} \in (\pi_m, \pi_i)$. Since both $G_m^B(l) - G_m^B(h) < c$ and $G_i^B(h) - G_i^B(l) < c$, if

$\epsilon > 0$ is sufficiently small, then $B + b_{mi}$ is IC and $\sigma(B + b_{mi}, \theta) = s(\theta)$ for every $\theta \in \Theta$. Both agents are better off adding b_{mi} to B since $-(1 - \pi_i)\epsilon - \pi_i \frac{\hat{\pi}-1}{\hat{\pi}}\epsilon > 0$ and $(1 - \pi_m)\epsilon + \pi_m \frac{\hat{\pi}-1}{\hat{\pi}}\epsilon > 0$. This contradicts the multilateral stability of B and, therefore, it cannot be that both $G_m^B(l) - G_m^B(h) < c$ and $G_i^B(h) - G_i^B(l) < c$. Since B is IC, it holds that $G_m^B(l) - G_m^B(h) \leq c$ and $G_i^B(h) - G_i^B(l) \leq c$. Hence, $G_m^B(l) - G_m^B(h) = c$ or $G_i^B(h) - G_i^B(l) = c$. By the same argument, $G_i^B(l) - G_i^B(h) = c$ or $G_k^B(h) - G_k^B(l) = c$.

If $G_k^B(h) - G_k^B(l) = c$ and $G_i^B(h) - G_i^B(l) = c$, then, by Lemma 1, B is not multilaterally stable. Also, if $G_i^B(h) - G_i^B(l) = -c$ and $G_m^B(h) - G_m^B(l) = -c$, then, by Lemma 1, B is not multilaterally stable. It follows that $G_k^B(h) - G_k^B(l) = c$ and $G_m^B(l) - G_m^B(h) = c$.

Since $n > 3$, there is an agent $i' \in I - \{i, m, k\}$ such that $\pi_i \neq \pi_{i'}$. If $G_i^B(h) - G_i^B(l) = 0 = G_{i'}^B(h) - G_{i'}^B(l)$, then we can construct a side-contract $b_{ii'}$ (similar to b_{mi} described above) that makes both i and i' better off. However, since B is multilaterally stable, it is impossible to construct such a side-contract and, therefore, $|G_i^B(h) - G_i^B(l)| > 0$ or $|G_{i'}^B(h) - G_{i'}^B(l)| > 0$. Recall that $G_k^B(h) - G_k^B(l) = c$ and $G_m^B(l) - G_m^B(h) = c$. By Lemma 1, this violates the multilateral stability of B.

To complete the proof, consider the case where $|I - M| > 1$. Observe that if there exist two agents $i, m \notin M$ such that $\pi_i > \pi_m$, then there exists a side-contract b_{mi}, similar to the side-contract b_{mi} described above, such that $B + b_{mi}$ is IC, $B + b_{mi} \succ_i B$, and $B + b_{mi} \succ_m B$. Hence, B is not multilaterally stable.

Proof of Proposition 2

Assume to the contrary that B is IC and multilaterally stable. By nontriviality, there exist two agents $m, m' \in M$ such that $w_m(H) - w_m(L) \geq c$ and $w_{m'}(L) - w_{m'}(H) \geq c$. If $G_m^B(l) - G_m^B(h) < c$ and $G_{m'}^B(h) - G_{m'}^B(l) < c$, then

$$w_m(H) + G_m^B(h) - w_m(L) - G_m^B(l) > 0 > w_{m'}(H) + G_{m'}^B(h) - w_{m'}(L) - G_{m'}^B(l).$$

In this case, m and m' would be better off signing a side-contract $b_{mm'}$ in which they provide each other with fair insurance (the stakes of the contract $b_{mm'}$ can be set to be small such that $B + b_{mm'}$ is IC and there is no manipulation). It follows that if B is IC and multilaterally stable, then $G_m^B(l) - G_m^B(h) = c$ or $G_{m'}^B(h) - G_{m'}^B(l) = c$.

Assume that $G_m^B(l) - G_m^B(h) = c$ (the case of $G_{m'}^B(h) - G_{m'}^B(l) = c$ is symmetric and omitted for brevity). By Lemma 1, if there exists an agent $i \neq m$ such that $G_i^B(l) > G_i^B(h)$, then B is not multilaterally stable. Thus, $G_i^B(l) \leq G_i^B(h)$ for each

$i \in I - \{m\}$. By richness, there exists an agent $i \notin M$ such that $w_i(H) - w_i(L) > 0$. As $G_i^B(l) \leq G_i^B(h)$, it follows that $w_i(H) + G_i^B(h) - w_i(L) - G_i^B(l) > 0$. If there exists an agent $j \notin M$ such that $w_j(H) + G_j^B(h) - w_j(L) - G_j^B(l) < 0$, then i and j are better off writing a side-contract b_{ij} in which they provide each other with fair insurance (note that $PM(B + b_{ij}) = \emptyset$ since $i, j \notin M$). As B is multilaterally stable, there exists no such agent $j \notin M$. By richness, there exists an agent $j \notin M$ such that $w_j(L) > w_j(H)$. Since $w_j(H) + G_j^B(h) - w_j(L) - G_j^B(l) \geq 0$, it follows that $G_j^B(h) > G_j^B(l)$.

By nontriviality, there exists an agent $m' \in M - \{m\}$ such that $w_{m'}(L) - w_{m'}(H) \geq c$. Since there exists an agent $i \notin M$ such that $w_i(H) + G_i^B(h) - w_i(L) - G_i^B(l) > 0$, the multilateral stability of B implies that $G_{m'}^B(h) - G_{m'}^B(l) = c$ (otherwise m' and i would be better off writing a side-contract in which they provide each other with fair insurance without violating the incentive compatibility of the collection of contracts).

In conclusion, there exist an agent $m \in M$ such that $G_m^B(l) - G_m^B(h) = c$, an agent $m' \in M$ such that $G_{m'}^B(h) - G_{m'}^B(l) = c$, and an agent $j \notin M$ such that $G_j^B(h) > G_j^B(l)$. By Lemma 1, this is in contradiction to the multilateral stability of B.

Proof of Proposition 3

In the first step of the proof, we construct a collection of contracts in which the agents' transfers sum up to zero. In the second step, we show that it is weakly stable.

For each $i \notin M$ and each pair of agents $j, k \in M$ set up three contracts $g^{i,j,k}$, $\hat{g}^{i,j,k}$, and $\tilde{g}^{i,j,k}$ such that

- $g_i^{i,j,k}(h) = -5c$
- $g_i^{i,j,k}(l) = 0$
- $\hat{g}_i^{i,j,k}(h) = 0$
- $\hat{g}_i^{i,j,k}(l) = -5c$
- $\tilde{g}_i^{i,j,k}(h) = 5c$
- $\tilde{g}_i^{i,j,k}(l) = 5c$
- $g_j^{i,j,k}(h) = -5c$
- $g_j^{i,j,k}(l) = 0$
- $\hat{g}_j^{i,j,k}(h) = 0$
- $\hat{g}_j^{i,j,k}(l) = 10c$
- $\tilde{g}_j^{i,j,k}(h) = 5c$
- $\tilde{g}_j^{i,j,k}(l) = -10c$
- $g_k^{i,j,k}(h) = 10c$
- $g_k^{i,j,k}(l) = 0$
- $\hat{g}_k^{i,j,k}(h) = 0$
- $\hat{g}_k^{i,j,k}(l) = -5c$
- $\tilde{g}_k^{i,j,k}(h) = -10c$
- $\tilde{g}_k^{i,j,k}(l) = 5c$

Denote the collection of these contracts by B'. In addition, for every pair of agents $i, j \in M$ set four bilateral contracts $b_{ij}^1, b_{ij}^2, b_{ij}^3, b_{ij}^4$ such that $b_{ij}^1(h) = -b_{ij}^2(h) = b_{ij}^3(l) =$

$-b^4_{ij}(l) = 5c$ and $b^1_{ij}(l) = b^2_{ij}(l) = b^3_{ij}(h) = b^4_{ij}(h) = 0$. Denote the collection of these contracts by B'' and let $B = B' + B''$. Note that $G^B_{i'}(s) = 0$ for every $s \in \{h, l\}$ and $i' \in I$. Thus, the collection B is IR and IC. We now show that B is also weakly stable.

We start by showing that no agent wishes to cancel a contract unilaterally. Consider three agents $i \notin M$ and $j, k \in M$ and a contract $g^{i,k,j} \in B'$. It is possible to see that $PM(B - g^{i,k,j}) = \{j, k\}$ and that, since $|G^B_k(h) - G^B_k(l) - g^{i,k,j}_k(h) + g^{i,k,j}_k(l)| = 10c > 5c = |G^B_j(h) - G^B_j(l) - g^{i,k,j}_j(h) + g^{i,k,j}_j(l)|$, Assumption 2 implies that $\sigma(B - g^{i,k,j}, \theta) = l$ for every $\theta \in \Theta$. Thus, no agent benefits from unilaterally cancelling this contract even if we do not take the cost of setting the signal into account. The analysis of unilaterally cancelling $\hat{g}^{i,j,k}$ is symmetric and therefore omitted. Suppose that $\tilde{g}^{i,j,k}$ is cancelled. It is possible to see that $PM(B - \tilde{g}^{i,k,j}) = \{j, k\}$ and that, since $|G^B_k(h) - G^B_k(l) - \tilde{g}^{i,k,j}_k(h) + \tilde{g}^{i,k,j}_k(l)| = 15c = 15c = |G^B_j(h) - G^B_j(l) - g^{i,k,j}_j(h) + g^{i,k,j}_j(l)|$, Assumption 2 implies that both agents obtain a net transfer of $-5c$ regardless of the state under $B - \tilde{g}^{i,k,j}$. Clearly, agent i obtains a transfer of $-5c$ regardless of the state under $B - \tilde{g}^{i,k,j}$. Thus, no agent is better off cancelling a contract in B' unilaterally.

Consider two manipulators, i and j, and a contract $b^z_{ij} \in B''$ where $z \in \{1, 2, 3, 4\}$. Clearly, $PM(B - b^z_{ij}) = \{i, j\}$. By Assumption 2, under $B - b^z_{ij}$ an agent who obtains his preferred realisation incurs a cost of $5c$. Thus, neither of the two agents is strictly better off cancelling b^z_{ij} unilaterally. We can conclude that no agent wishes to cancel a contract unilaterally.

To complete the proof, we show that for every side-contract $g^K \notin B$, if $B + g^K \succ_i B$ for some $i \in K$, then there exists an agent $j \in K - \{i\}$ and a permissible conjecture $\beta_j(g^K, i)$ that blocks g^K. Consider a side-contract g^K such that $PM(B + g^K) \neq \emptyset$. By Assumptions 1 and 2, either $\sigma(B + g^K, \theta) = l$ for every $\theta \in \Theta$ or $\sigma(B + g^K, \theta) = h$ for every $\theta \in \Theta$. Assume that $\sigma(B + g^K, \theta) = l$ for every $\theta \in \Theta$ (the second case is analogous and omitted for brevity). Since $\beta_{k^\star}(g^K, i) = \emptyset$ is permissible, if there is an agent $k^\star \in K - \{i\}$ such that $g^K_{k^\star}(l) < 0$, then we have found a permissible conjecture that blocks g^K. Suppose that $g^K_{k^\star}(l) \geq 0$ for every $k^\star \in K - \{i\}$. Since g^K is budget-balanced, it must be that $g^K_i(l) \leq 0$, in contradiction to $B + g^K \succ_i B$.

Consider a side-contract g^K such that $PM(B + g^K) = \emptyset$. If $B + g^K \succ_i B$, then $g^K_i(l) > 0$ or $g^K_i(h) > 0$. Without loss of generality, assume that $g^K_i(l) > 0$ and $g^K_i(l) \geq g^K_i(h)$. Since g^K is budget-balanced, there exists an agent $k^\star \in K$ such that $g^K_{k^\star}(l) < 0$. If $B \succ_{k^\star} B + g^K$, then the permissible conjecture $\beta_{k^\star}(g^K, i) = \emptyset$ blocks the side-contract. Suppose that $B \not\succ_{k^\star} B + g^K$. It follows that $g^K_{k^\star}(h) > 0 > g^K_{k^\star}(l)$. We now find a permissible conjecture for $k^\star$ that blocks g^K. Split the analysis into two cases: (1) $i \notin M$ and (2) $i \in M$.

Case (1). Consider a conjecture $\beta_{k^\star}(g^K, i) = -g^{i,j,k}$ as described above (since $|M| \geq 3$, it follows that $|M - \{k^\star\}| \geq 2$ and we can assume that $k^\star \notin \{j, k\}$). Since $PM(B + g^K) = \emptyset$, it follows that $PM(B - g^{i,j,k}) = PM(B + g^K - g^{i,j,k}) = \{j, k\}$. Moreover, by Assumption 2, $\sigma(B + g^K - g^{i,j,k}, \theta) = \sigma(B - g^{i,j,k}, \theta) = l$ for every $\theta \in \Theta$. Hence, under the conjecture agent i obtains $g_i^K(l) > 0$ regardless of the state if g^K is signed and 0 if it is not signed. Thus, the conjecture is consistent. It blocks the deviation since $g_{k^\star}^K(l) < 0$ and it is permissible since it does not require the consent of a third party.

Case (2.i). Let $I = M$. Recall that $g_{k^\star}^K(h) > 0 > g_{k^\star}^K(l)$ and $g_i^K(h) \leq g_i^K(l)$. Since g^K is budget-balanced, there is an agent $j \in K - \{i, k^\star\}$ such that $G_j^B(h) + g_j^K(h) - G_j^B(l) - g_j^K(l) < G_i^B(l) + g_i^K(l) - G_i^B(h) - g_i^K(h)$ or there is an agent $j \in M - K$ and $g_i^K(h) < g_i^K(l)$. In either case, consider a conjecture $\beta_{k^\star}(g^K, i) = -b_{ij}^2$. Since $PM(B + g^K) = \emptyset$, it follows that $PM(B - b_{ij}^2) = PM(B + g^K - b_{ij}^2) = \{j, i\}$. Moreover, by Assumption 2, $\sigma(B + g^K - b_{ij}^2, \theta) = \sigma(B - b_{ij}^2, \theta) = l$ for every $\theta \in \Theta$ and agent i does not incur any cost to set the signal in $B - b_{ij}^2$ or in $B - b_{ij}^2 + g^K$. Hence, under this conjecture, in both states, agent i obtains a transfer of 0 if g^K is not signed and a transfer of $g_i^K(l) > 0$ if it is signed. Thus, the conjecture is consistent. The conjecture blocks g^K since $g_{k^\star}^K(l) < 0$ and it is permissible since it does not require the consent of a third party.

Case (2.ii). Let $|I| = |M| + 1$. If $k^\star \notin M$, then we can use the proof of case (2.i). Suppose that $k^\star \in M$ and consider a conjecture $\beta_{k^\star}(g^K, i) = -g^{i',i,k}$ as described above (with $i' \notin M$, $i = j$, and $k \in M - \{k^\star\}$). Since $PM(B + g^K) = \emptyset$, it follows that $PM(B - g^{i',i,k}) = PM(B + g^K - g^{i',i,k}) = \{i, k\}$. Moreover, by Assumption 2, $\sigma(B + g^K - g^{i',i,k}, \theta) = \sigma(B - g^{i',i,k}, \theta) = l$ for every $\theta \in \Theta$. Under $B - g^{i',i,k}$ agent i obtains 0 regardless of the state while under $B - g^{i',i,k} + g^K$ he obtains at least $g_i^K(l) > 0$ regardless of the state. Thus, the conjecture is consistent. It blocks the deviation since $g_{k^\star}^K(l) < 0$ and it is permissible since it does not require the consent of a third party.

Case (2.iii). Let $|I| > |M| + 1$. In this case, there are two agents $i' \notin M$ and $k \in M$ such that $i' \neq k^\star$ and $k \neq k^\star$. Thus, we can apply the argument in the proof of Case (2.ii) to show that the conjectured $\beta_{k^\star}(g^K, i)$ that is described above is permissible and blocks g^K.

We can conclude that for every side-contract g^K that makes one of the members of K better off, at least one of the counterparties to the deviation has a permissible conjecture that makes him worse off. Thus, B is weakly stable.

Proof of Proposition 4

The proof relies on the following lemma.

Lemma 2 *Suppose that B is IC and consider a side-contract b_{mi} such that $i \notin M$, $b_{mi}(h), b_{mi}(l) \geq 0$, and $PM(B + b_{mi}) = \{m\}$. There exists no consistent conjecture $\beta_m(b_{mi}, i)$ that blocks b_{mi}.*

Proof. We will show that every conjecture $\beta_m(b_{mi}, i)$ that blocks b_{mi} is not consistent. Assume that $b_{mi}(h) > b_{mi}(l)$. Note that under B agent m's expected payoff is

$$\pi_m u_m(w_m(H) + G_m^B(h)) + (1 - \pi_m)u_m(w_m(L) + G_m^B(l)). \tag{2}$$

Since $PM(B + b_{mi}) = \{m\}$ and $b_{mi}(h) > b_{mi}(l)$, it follows that $G_m^B(h) + b_{mi}(h) - (G_m^B(l) + b_{mi}(l)) > c$. By Assumption 1, under $B + b_{mi}$ agent m's expected payoff is

$$\pi_m u_m(w_m(H) + G_m^B(h) + b_{mi}(h)) + (1 - \pi_m)u_m(w_m(L) + G_m^B(h) + b_{mi}(h) - c). \tag{3}$$

Note that $b_{mi}(l) \geq 0$ implies that $G_m^B(h) + b_{mi}(h) - c > G_m^B(l)$. Hence, (3) is strictly greater than (2).

We now show that there exists no consistent conjecture $\beta_m(b_{mi}, i)$ that blocks b_{mi}. Consider a conjecture $\beta_m(b_{mi}, i)$. First, if $PM(B + \beta_m(b_{mi}, i)) = \emptyset$, then m obtains an expected payoff as in (3) if he signs b_{mi} and an expected payoff as in (2) otherwise. Hence, $B + b_{mi} + \beta_m(b_{mi}, i) \succ_m B + \beta_m(b_{mi}, i)$. Second, suppose that $PM(B + \beta_m(b_{mi}, i)) \neq \emptyset$. Assumptions 1 and 2 imply that either (i) $\sigma(B + \beta_m(b_{mi}, i), \theta) = h$ for every $\theta \in \Theta$ or (ii) $\sigma(B + \beta_m(b_{mi}, i), \theta) = l$ for every $\theta \in \Theta$.

In case (i), Assumptions 1 and 2 imply that $\sigma(B + \beta_m(b_{mi}, i), \theta) = h = \sigma(B + \beta_m(b_{mi}, i) + b_{mi}, \theta)$ for every $\theta \in \Theta$. Thus, agent i is worse off signing b_{mi} given $\beta_m(b_{mi}, i)$ (given m's conjecture, the side-contract b_{mi} is essentially a positive transfer from i to m that has no effect on the signal). Hence, $\beta_m(b_{mi}, i)$ is not consistent.

In case (ii), agent m obtains an expected payoff of

$$\pi_m u_m(w_m(H) + G_m^B(l)) + (1 - \pi_m)u_m(w_m(L) + G_m^B(l)) \tag{4}$$

if he does not sign b_{mi} and an expected payoff of at least

$$\pi_m u_m(w_m(H) + G_m^B(l) + b_{mi}(l)) + (1 - \pi_m)u_m(w_m(L) + G_m^B(l) + b_{mi}(l)) \tag{5}$$

if he does sign b_{mi}. Since $b_{mi}(l) \geq 0$, the conjecture $\beta_m(b_{mi}, i)$ does not block b_{mi}.

Note that we assumed throughout the proof that $b_{mi}(h) > b_{mi}(l)$. The case of $b_{mi}(h) < b_{mi}(l)$ is symmetric and omitted for brevity. Finally, note that $b_{mi}(h) = b_{mi}(l)$ together with $PM(B) = \emptyset$ contradicts $PM(B + b_{mi}) = \{m\}$. ■

We now complete the proof of Proposition 4. The first step shows that if richness and nontriviality are satisfied and B is constrained-efficient, then there exist an agent $m \in M$ such that $\left|G_m^B(h) - G_m^B(l)\right| = c$ and an agent $i \notin M$ such that $sign\left(G_m^B(h) - G_m^B(l)\right) = sign\left(G_i^B(h) - G_i^B(l)\right)$.

Step 1. Nontriviality implies that there exist two agents $m, m' \in M$ such that $w_m(H) - w_m(L) \geq c > -c \geq w_{m'}(H) - w_{m'}(L)$. If $G_m^B(l) - G_m^B(h) < c$ and $G_{m'}^B(h) - G_{m'}^B(l) < c$, then m and m' can write a side-contract $b_{mm'}$ in which they provide each other with fair insurance such that $PM(B + b_{mm'}) = \emptyset$. Both m and m' are better off signing this side-contract. Note that the constrained efficiency of B implies that there exists no IC collection $B+b_{mm'}$ such that $B+b_{mm'} \succ_{m'} B$, and $B+b_{mm'} \succ_m B$. It follows that if B is constrained-efficient, then $G_m^B(l) - G_m^B(h) = c$ or $G_{m'}^B(h) - G_{m'}^B(l) = c$.

Without loss of generality, assume that $G_m^B(l) - G_m^B(h) = c$. If there exists an agent $i \notin M$ such that $G_i^B(l) > G_i^B(h)$, then we have found a pair of agents, $m \in M$ and $i \notin M$, such that $|G_m^B(h) - G_m^B(l)| = c$ and $sign\left(G_m^B(h) - G_m^B(l)\right) = sign\left(G_i^B(h) - G_i^B(l)\right)$. Suppose that for each $i \notin M$, $G_i^B(h) \geq G_i^B(l)$.

By richness, there exists an agent $i \notin M$ such that $w_i(H) > w_i(L)$. Since $G_i^B(h) \geq G_i^B(l)$, it follows that $w_i(H) + G_i^B(h) - w_i(L) - G_i^B(l) > 0$. If there exists an agent $j \notin M$ such that $w_j(H) + G_j^B(h) - w_j(L) - G_j^B(l) < 0$, then i and j are better off writing a side-contract b_{ij} in which they provide each other with fair insurance. The existence of the collection $B + b_{ij}$ violates the constrained efficiency of B. Hence, if B is constrained-efficient, then $w_j(H) + G_j^B(h) - w_j(L) - G_j^B(l) \geq 0$ for every agent $j \in I - M$. By richness, there exists an agent $j \in I - M$ such that $w_j(L) > w_j(H)$. Thus, if B is constrained-efficient, then $G_j^B(h) > G_j^B(l)$.

By nontriviality, there exists an agent $m' \in M - \{m\}$ such that $w_{m'}(L) - w_{m'}(H) \geq c$. Recall that there exists an agent $i \notin M$ such that $w_i(H) + G_i^B(h) - w_i(L) - G_i^B(l) > 0$. If B is constrained-efficient, then there exists no side-contract $b_{im'}$ such that: (i)

$B + b_{im'}$ is IC, (ii) $B + b_{im'} \succ_i B$, and (iii) $B + b_{im'} \succ_{m'} B$. If $G^B_{m'}(h) - G^B_{m'}(l) < c$, there is always a side-contract $b_{im'}$ in which both parties provide each other with fair insurance that satisfies (i), (ii), and (iii). Thus, the constrained efficiency of B implies that $G^B_{m'}(h) - G^B_{m'}(l) = c$. Since we already found an agent $j \notin M$ such that $G^B_j(h) > G^B_j(l)$, the first part of the proof is complete.

Step 2. Suppose that B is constrained-efficient and, therefore, IC. Without loss of generality, assume that there exist an agent $m \in M$ such that $G^B_m(l) - G^B_m(h) = c$ and an agent $i \notin M$ such that $G^B_i(l) > G^B_i(h)$. Consider a side-contract b_{mi} such that $b_{mi}(l) > b_{mi}(h) > 0$. Since B is IC and $G^B_m(l) - G^B_m(h) + b_{mi}(l) - b_{mi}(h) > c$, it follows that $PM(B + b_{mi}) = \{m\}$. By Assumption 1, $\sigma(B + b_{mi}, \theta) = l$ for each $\theta \in \{L, H\}$. If $b_{mi}(l)$ is sufficiently close to 0, then

$$\pi u_i(w_i(H) + G^B_i(h)) + (1-\pi)\, u_i\left(w_i(L) + G^B_i(l)\right) <$$
$$\pi u_i\left(w_i(H) + G^B_i(l) - b_{mi}(l)\right) + (1-\pi)\, u_i\left(w_i(L) + G^B_i(l) - b_{mi}(l)\right)$$

and, therefore, $B + b_{mi} \succ_i B$. By Lemma 2, there exists no consistent conjecture $\beta_m(b_{mi}, i)$ that blocks b_{mi}. Thus, there exists no permissible conjecture $\beta_m(b_{mi}, i)$ that blocks b_{mi}, and so B is not weakly stable.

Proof of Proposition 5

Consider an IC collection B. Suppose that there exists an agent $i \in M$ such that $G^B_i(l) - G^B_i(h) < c$ or an agent $i \in L - M$ such that $G^B_i(l) - G^B_i(h) < w$. Consider a side-contract b_{ij} such that $b_{ij}(l) = \epsilon > 0$, $b_{ij}(h) = -\frac{1-\pi}{\pi}\epsilon$, and $j \in E$. If ϵ is sufficiently small, then $B + b_{ij}$ is IC. Since $j \in E$ is risk-neutral and he provides fair coverage to i in the contract b_{ij}, it follows that $B + b_{ij} \succ_i B$ and $B \not\succ_j B + b_{ij}$. This is in contradiction to B being constrained-efficient. Thus, B is constrained-efficient only if $G^B_i(l) - G^B_i(h) = c$ for every $i \in M$ and $G^B_i(l) - G^B_i(h) \geq w$ for every $i \in L - M$. To complete the proof, we need only repeat Step 2 in the proof of Proposition 4.

Proof of Proposition 6

Step 1 considers one type of side-contract and shows that, if it makes both parties better off, then it violates the pairwise weak stability of the collection of contracts.

Step 1. For any collection of contracts B and agent $i \in L - M$, let

$$z_i\left(G_i^B(l) - G_i^B(h)\right) = \frac{1}{\gamma} log\left[\frac{\pi exp[\gamma\left(G_i^B\left(l\right) - G_i^B\left(h\right)\right)] + \left(1-\pi\right) exp[\gamma\left(w_i\left(H\right) - w_i\left(L\right)\right)]}{\pi + \left(1-\pi\right) exp[\gamma\left(w_i\left(H\right) - w_i\left(L\right)\right)]}\right] \tag{6}$$

be agent i's willingness to pay to guarantee that, ex post, the signal will be l, that is,

$$\pi u(w_i(H) + G_i^B(l) - z_i(G_i^B\left(l\right) - G_i^B(h))) + (1-\pi)u(w_i(L) + G_i^B(l) - z_i(G_i^B\left(l\right) - G_i^B(h))) \\ = \pi u(w_i(H) + G_i^B(h)) + (1-\pi)u(w_i(L) + G_i^B(l)).$$

Consider an IC collection B and a pair of agents, $i \in L - M$ and $j \in M$, such that $z_i(G_i^B\left(l\right) - G_i^B(h)) + G_j^B(l) - G_j^B(h) > c$. This implies that $z_i(G_i^B\left(l\right) - G_i^B(h)) > 0$. Suppose that i and j write a side-contract b_{ji} such that $0 < \epsilon = b_{ji}(h) < b_{ji}(l) = z_i(G_i^B\left(l\right) - G_i^B(h)) - \epsilon$. If ϵ is sufficiently small, then $PM(B + b_{ij}) = \{j\}$ and, by Assumption 1, $\sigma(B + b_{ij}, \theta) = l$ for every $\theta \in \Theta$. As $z_i(G_i^B\left(l\right) - G_i^B(h)) > 0$, it follows that $G_i^B\left(l\right) - G_i^B(h) > 0$. Thus, if ϵ is sufficiently small, then $B + b_{ji} \succ_i B$. By Lemma 2, b_{ji} is not blocked by any consistent conjecture $\beta_j(b_{ji})$ (and, hence, by no permissible conjecture). Thus, B is pairwise weakly stable only if, for every $i \notin M$ and $j \in M$,

$$z_i(G_i^B\left(l\right) - G_i^B(h)) + G_j^B(l) - G_j^B(h) \leq c. \tag{7}$$

Step 2. We now study the coverage maximisation problem subject to (7) and the incentive-compatibility constraint. Note that $z_i(\cdot)$ depends only on the size of i's exposure $G_i^B(l) - G_i^B(h)$. Thus, we can restrict attention to collections B where $G_i^B(l) - G_i^B(h) = G_j^B(l) - G_j^B(h)$ for any pair of agents $i, j \in J \in \{L - M, M\}$ (if there are two agents $i, j \in J$ with asymmetric exposure it is always possible to enlarge the exposure of one of these agents by setting a bilateral contract with a member of E that does not violate any of the constraints). For every such symmetric collection B, denote by $R_m\left(B\right) := G_m^B\left(l\right) - G_m^B\left(h\right)$ and $R_k\left(B\right) := G_k^B\left(l\right) - G_k^B\left(h\right)$ the coverage provided to each $m \in M$ and $k \in L - M$, respectively. When there is no risk of confusion, we omit B and write R_m, R_k, and $z(R_k)$. Thus, we can write (7) as

$$z(R_k) + R_m \leq c. \tag{8}$$

At the optimum, (8) must hold in equality as, otherwise, we could increase R_m, thereby strictly increasing the agents' coverage.

Denote the solution of the coverage maximisation problem given α by R_m^α and R_k^α. The aggregate coverage is maximised if and only if the average coverage $\alpha R_m + (1 - \alpha) min\{R_k, w\}$ is maximised. Note that (8) is concave due to the CARA assumption. Thus, the maximisation problem is equivalent to maximising a convex combination $\alpha R_m + (1-\alpha)R_k$ subject to a concave constraint and to the requirements that $R_m \in [-c, c]$ and $R_k \leq w$. It follows that R_m^α (resp., R_k^α) is increasing (resp., decreasing) in α and there exists $\alpha^\star \in (0,1)$ such that (i) $R_m^{\alpha^\star} = R_k^{\alpha^\star}$ and (ii) the aggregate coverage is increasing in α for $\alpha < \alpha^\star$ and decreasing in α for $\alpha > \alpha^\star$.

Fix an arbitrary $\alpha \in (0,1)$. First, note that if $R_k^\alpha < 0$ and $R_m^\alpha \leq c$, then (8) is slack. Thus, $R_k^\alpha \geq 0$. Second, note that $R_k^\alpha \leq w$. By our assumption on the size of w, it holds that $z(w) \leq c$. Hence, $z(R_k^\alpha) \leq c$, and so $R_m^\alpha \geq 0$. We can conclude that $R_m^\alpha \geq 0$ and $R_k^\alpha \geq 0$ for every $\alpha \in (0,1)$. Clearly $R_m^\alpha = c > 0$ for $\alpha = 1$ as $L = M$ in this case.

Step 3. Fix an arbitrary α and let $R_k^\star = R_k^\alpha$ and $R_m^\star = R_m^\alpha$. We now construct an IR and pairwise weakly stable collection of contracts that induces $R_k^\star$ and $R_m^\star$.

Consider a multilateral contract $\hat{g}$ in which $\hat{g}_i(h) - \hat{g}_i(l) = (\alpha R_m^\star + (1-\alpha)R_k^\star)\frac{|L|}{|E|}$ for every $i \in E$, $\hat{g}_i(l) - \hat{g}_i(h) = R_m^\star$ for every $i \in M$, and $\hat{g}_i(l) - \hat{g}_i(h) = R_k^\star$ for every $i \in L - M$. Moreover, set the contract such that each $i \in E$ provides fair coverage (i.e., $\pi\hat{g}_i(h) + (1-\pi)\hat{g}_i(l) = 0$) and each $i \in L$ obtains fair coverage.

For each $i \in I$ let z_i be implicitly defined by

$$\begin{aligned} &\pi u_i(w_i(H) + \hat{g}_i(l) - z_i) + (1-\pi)u_i(w_i(L) + \hat{g}_i(l) - z_i) \\ &\quad = \pi u_i(w_i(H) + \hat{g}_i(h)) + (1-\pi)u_i(w_i(L) + \hat{g}_i(l)) \end{aligned}$$

and let q_i be implicitly defined by

$$\begin{aligned} &\pi u(w_i(H) + \hat{g}_i(h) - q_i) + (1-\pi)u(w_i(L) + \hat{g}_i(h) - q_i) \\ &\quad = \pi u(w_i(H) + \hat{g}_i(h)) + (1-\pi)u(w_i(L) + \hat{g}_i(l)). \end{aligned}$$

Since $R_m^\star \geq 0$ it follows that $z_i \in [0, c)$ and $q_i \in (-c, 0]$ for every $i \in M$. Choose an agent $i \in I$ and three agents, $z \in I - M$ and $j, k \in M$. Set up two contracts as follows.

- $g_i^{i,j,k,z}(h) = -q_i$
- $\overline{g}_i^{i,j,k,z}(h) = q_i$
- $g_j^{i,j,k,z}(h) = -10c$
- $g_i^{i,j,k,z}(l) = z_i$
- $\overline{g}_i^{i,j,k,z}(l) = -z_i$
- $g_j^{i,j,k,z}(l) = 10c$

- $\overline{g}_j^{i,j,k,z}(h) = 10c$
- $\overline{g}_j^{i,j,k,z}(l) = -10c$
- $g_k^{i,j,k,z}(h) = 15c$
- $g_k^{i,j,k,z}(l) = -15c$
- $\overline{g}_k^{i,j,k,z}(h) = -15c$
- $\overline{g}_k^{i,j,k,z}(l) = 15c$
- $g_z^{i,j,k,z}(h) = q_i - 5c$
- $g_z^{i,j,k,z}(l) = 5c - z_i$
- $\overline{g}_z^{i,j,k,z}(h) = 5c - q_i$
- $\overline{g}_z^{i,j,k,z}(l) = z_i - 5c$

Repeat this process twice (each time with a different set of three agents) for each agent $i \in I$ and denote the collection of these contracts by B'. Note that the transfers in B' sum up to zero and that, in $\hat{g}$, risk-neutral agents provide fair coverage to risk-averse agents. Thus, the collection $B = B' + \hat{g}$ is IR.

Consider an arbitrary group of four agents i, j, k, z who signed two contracts as described above. We show that none of them can benefit from unilaterally cancelling one of these contracts. By Assumption 2, $\sigma(B - g^{i,j,k,z}, \theta) = l$ and $\sigma(B - \overline{g}^{i,j,k,z}, \theta) = h$ for every $\theta \in \Theta$. By the definition of z_i and q_i, agent i is indifferent whether to cancel one of these contracts or not. Agents j and z obtain a strictly negative transfer regardless of the state if they cancel one of these contracts. Agent k obtains a positive transfer if he cancels one of these contracts. However, due to agent k's cost of setting the signal to his preferred realisation, he is worse off unilaterally cancelling one of these contracts.

To complete the proof, we need to show that for any agent i and side-contract b_{ix} such that $B + b_{ix} \succ_i B$, there exists a permissible conjecture $\beta_x(b_{ix})$ that blocks it. Note first that if $B \succ_x B + b_{ix}$, then the (permissible) conjecture $\beta_x(b_{ix}) = \emptyset$ blocks the deviation. We therefore focus on cases where $B + b_{ix} \succ_i B$ and $B \not\succ_x B + b_{ix}$.

There are four cases to consider: (1) $PM(B + b_{ix}) = \emptyset$, (2) $PM(B + b_{ix}) = \{i\}$, (3) $PM(B + b_{ix}) = \{x\}$, and (4) $PM(B + b_{ix}) = \{i, x\}$.

Case 1. Since $B + b_{ix} \succ_i B$ and $B \not\succ_x B + b_{ix}$, it must be that $b_{ix}(h) > 0 > b_{ix}(l)$ or $b_{ix}(h) < 0 < b_{ix}(l)$. Let $b_{ix}(h) < 0 < b_{ix}(l)$ and consider a conjecture $\beta_x(b_{ix}) = -g^{i,j,k,z}$. Since $PM(B + b_{ix}) = \emptyset$, it follows that $b_{ix}(l) - b_{ix}(h) \leq c - R_m$ if $i \in M$. Thus, $i \notin PM(B + b_{ix} - g^{i,j,k,z})$. Moreover, $PM(B + b_{ix}) = \emptyset$ implies that $x \notin PM(B + b_{ix} - g^{i,j,k,z})$. Hence, $PM(B + b_{ix} - g^{i,j,k,z}) = (B - g^{i,j,k,z}) = \{j, k\}$ and, by Assumption 2, $\sigma(B - g^{i,j,k,z}, \theta) = l = \sigma(B - g^{i,j,k,z} + b_{ix}, \theta)$ for every $\theta \in \Theta$. Thus, under $\beta_x(b_{ix})$, the side-contract b_{ix} is a state-independent transfer of $b_{ix}(l)$ from x to i. Hence, $\beta_x(b_{ix})$ is consistent and blocks b_{ix}. Clearly, $\beta_x(b_{ix}) \in A_x^0(B, b_{ix}, i)$ and so it is permissible. The analysis of the case where $b_{ix}(h) > 0 > b_{ix}(l)$ is similar and omitted for brevity.

Case 2. Suppose that $b_{ix}(l) > b_{ix}(h)$. Since $B + b_{ix} \succ_i B$, it follows that $b_{ix}(l) > 0$.

If $|L-M|>1$, consider a conjecture $\beta_x(b_{xi})=b_{ji}$ such that $j\in L-M-\{x\}$, $b_{ji}(l)=0$, and $b_{ji}(h)=c+\epsilon$. Note that $PM(B+b_{ji})=PM(B+b_{xi}+b_{ji})=\{i\}$. By Assumption 1, $\sigma(B+b_{xi}+b_{ji},\theta)=l=\sigma(B+b_{ji},\theta)$ and so $\beta_x(b_{ix})=b_{ji}$ is consistent and blocks b_{ix}. The conjecture is permissible as there is no conjecture $\beta_j(b_{ji})$ that blocks b_{ji}. To see that no conjecture can block b_{ji}, consider a conjecture $\beta_j(b_{ji})$. If $\sigma(B+\beta_j(b_{ji}),\theta)=\sigma(B+\beta_j(b_{ji})+b_{ji},\theta)$ for every $\theta\in\Theta$, then $\beta_j(b_{ji})$ does not block b_{ji}. If $\sigma(B+\beta_j(b_{ji}),\theta)\neq\sigma(B+\beta_j(b_{ji})+b_{ji},\theta)$, then $\sigma(B+\beta_j(b_{ji}),H)=h$ and $\sigma(B+\beta_j(b_{ji})+b_{ji},L)=l$ and, since $R^\star_k\geq 0$, the conjecture does not block b_{ji}.

If $L-M\leq 1$, then $R^\star_m>0$. We can assume that $j\in M-\{x\}$, $b_{ji}(l)=0$, and $b_{ji}(h)=c$, and repeat the exercise in the paragraph above.

The case where $b_{ix}(h)>b_{ix}(l)$ is symmetric (the analysis is essentially identical except that we choose $j\in E$ instead of $j\in L$) and is omitted for brevity. Note that if $b_{ix}(h)=b_{ix}(l)$, then it contradicts $i\in PM(B)$.

Case 3. For convenience, we write b_{xi} instead of b_{ix} in this case. We need to split the analysis into two subcases: (a) $i\notin M$ and (b) $i\in M$.

First, let $i\notin M$. Assume that $b_{xi}(l)>b_{xi}(h)$. By Assumption 1, $\sigma(B+b_{xi},\theta)=l$ for every $\theta\in\Theta$. As $B\not\succ_x B+b_{xi}$, it follows that $b_{xi}(l)>0$. As $B+b_{xi}\succ_i B$, it follows that $i\in L-M$ and $b_{xi}(l)<z(R_k)=z_i$. By (8), $PM(B+b_{xi})=\{x\}$ implies that $b_{xi}(h)<0$. Consider a conjecture $\beta_x(b_{ix})=b_{ji}$ such that $j\in M-\{x\}$, $b_{ji}(h)=R^\star_m+(b_{xi}(l)-b_{xi}(h)+R^\star_m)-\epsilon$, and $b_{ji}(l)=0$. By Lemma 2, this conjecture is not blocked by any consistent conjecture $\beta_j(b_{ji})$. If $\epsilon>0$ is sufficiently small, then, by Assumption 2, $\sigma(B+b_{xi}+b_{ji},\theta)=l$ and $\sigma(B+b_{ji},\theta)=h$ for every $\theta\in\Theta$. Thus, $B+b_{xi}+b_{ji}\sim_i B+b_{xi}\succ_i B\succ_i B+b_{ji}$. Hence, $\beta_j(b_{ji})$ is consistent (and, since it is not blocked by any consistent conjecture, it is also permissible). Finally, under this conjecture, agent x obtains a net transfer of $G^B_i(h)+b_{xi}(h)+\epsilon$ in both states. Hence, if ϵ is sufficiently small, the deviation is blocked by the conjecture.

The analysis of the case where $b_{xi}(l)<b_{xi}(h)$ is symmetric and omitted for brevity. Note that $b_{xi}(l)=b_{xi}(h)$ violates the assumption that $x\in PM(B+b_{xi})$.

Second, let $i\in M$. Note that $PM(B+b_{xi})=\{x\}$ implies that $R^\star_m>0$ and that $\sigma(B+b_{xi},\theta)=l$ for every $\theta\in\Theta$. Since $B+b_{xi}\succ_i B$, it follows that

$$\begin{aligned}\pi u_i\left(w_i\left(H\right)+G_i^B\left(h\right)\right)+\left(1-\pi\right)u_i\left(w_i\left(L\right)+G_i^B\left(l\right)\right)&<\\ \pi u_i\left(w_i\left(H\right)+G_i^B\left(l\right)-b_{xi}\left(l\right)\right)+\left(1-\pi\right)u_i\left(w_i\left(L\right)+G_i^B\left(l\right)-b_{xi}\left(l\right)\right)&.\end{aligned}$$

Plugging constant absolute risk aversion into the above expression and rearranging, we obtain inequality (9), which provides an upper bound for $b_{xi}(l)$:

$$b_{xi}(l) < \frac{1}{\gamma} log \left[\frac{\pi exp[\gamma \left(G_i^B(l) - G_i^B(h) \right)] + (1-\pi) exp[\gamma (w_i(H) - w_i(L))]}{\pi + (1-\pi) exp[\gamma (w_i(H) - w_i(L))]} \right]. \tag{9}$$

We now find a permissible conjecture $\beta_x(b_{xi})$ to block b_{xi}. Consider $\beta_x(b_{xi}) = b_{ji}$, such that $j \in L$, $b_{ji}(l) = 0$, $G_i^B(l) - G_i^B(h) - b_{ji}(l) + b_{ji}(h) > c$, and $G_i^B(l) - G_i^B(h) - b_{ji}(l) + b_{ji}(h) - b_{xi}(l) + b_{xi}(h) \leq c$. Under this conjecture, the side-contract b_{xi} has no effect on the signals' distribution (agent i's incentive-compatibility constraint is violated under $B + b_{ji}$ and agent x's incentive-compatibility constraint is violated under $B + b_{ji} + b_{xi}$). Rather, it affects the identity of the agent who pays for manipulation as agent x is the one who incurs the cost of manipulation under $B + b_{ji} + b_{xi}$. Observe that the conjecture does not block the deviation if and only if

$$\begin{aligned}\pi u_x \left(w_x(H) + G_x^B(l) + b_{xi}(l) - c \right) + (1-\pi) u_x \left(w_x(L) + G_x^B(l) + b_{xi}(l) \right) \geq \\ \pi u_x \left(w_x(H) + G_x^B(l) \right) + (1-\pi) u_x \left(w_x(L) + G_x^B(l) \right).\end{aligned}$$

Plugging constant absolute risk aversion into the above expression, we obtain

$$b_{xi}(l) \geq \frac{1}{\gamma} log \left[\frac{\pi exp[\gamma c] + (1-\pi) exp[\gamma (w_x(H) - w_x(L))]}{\pi + (1-\pi) exp[\gamma (w_x(H) - w_x(L))]} \right]. \tag{10}$$

Since B is IC, $c \geq G_i^B(l) - G_x^B(h)$ and, therefore, there exists no contract b_{ix} that satisfies both (10) and (9). Inequality (11) guarantees that $\beta_x(b_{xi})$ is consistent:

$$\begin{aligned}\pi u_i \left(w_i(H) + G_i^B(l) + b_{ij}(l) - c \right) + (1-\pi) u_i \left(w_i(L) + G_i^B(l) + b_{ij}(l) \right) \\ < \pi u_i \left(w_i(H) + G_i^B(l) + b_{ij}(l) - b_{xi}(l) \right) \\ + (1-\pi) u_i \left(w_i(L) + G_i^B(l) + b_{ij}(l) - b_{xi}(l) \right).\end{aligned} \tag{11}$$

One can verify that inequality (11) is implied by inequality (9). Since j can only benefit from signing b_{ji}, $\beta_x(b_{xi})$ is not blocked by any conjecture $\beta_j(\beta_x(b_{xi}), i)$ and, therefore, it is permissible.

Case 4. If $R_m^\star = 0$, then $\sigma(B + b_{ix}) = h$ for every $\theta \in \Theta$. Hence, $B + b_{ix} \succ_i B$ implies that $B \succ_x B + b_{ix}$. In the remainder of the proof, we assume that $R_m^\star > 0$. When $R_m^\star > 0$ and $i, x \in PM(B + b_{ix})$, Assumption 2 implies that $\sigma(B + b_{ix}, \theta) = l$ for every $\theta \in \Theta$.

Assume that $b_{ix}(l) > b_{ix}(h)$. Since $B + b_{ix} \succ_i B$, it follows that $b_{ix}(l) > 0$. Consider a conjecture $\beta_x(b_{ix}) = b_{ji}$ such that $j \in L$, $b_{ji}(l) = 0$, and $b_{ji}(h) = c$. Note that $PM(B+b_{ji}) = \{i\}$ and that $\sigma(B+b_{ji}, \theta) = \sigma(B+b_{ji}+b_{ix}, \theta) = l$ for every $\theta \in \Theta$. Under this conjecture, b_{ix} is a positive transfer of $b_{ix}(l) > 0$ from x to i regardless of the state. Hence, the conjecture blocks the deviation. Since $B+b_{ji}+b_{ix} \sim_i B+b_{ix} \succ_i B \succeq_i B+b_{ji}$ the conjecture is consistent. Clearly, there is no conjecture that blocks it and, as a result, it is permissible.

Assume that $b_{ix}(l) < b_{ix}(h)$. Since $B + b_{ix} \succ_i B$, it follows that $-b_{ix}(l) < z_i$ (see the RHS of (9)). The rest of the proof is similar to the proof of Case 3 with $i \in M$ and is omitted for brevity. Note that $b_{ix}(l) = b_{ix}(h)$ violates the assumption that $i, x \in PM(B + b_{ix})$.

Proof of Proposition 7

The proof uses the same ideas and arguments as the proof of Proposition 6. We shall refer the reader to the proof of Proposition 6 whenever there is redundancy.

Step 1. For every collection B, let $z_i^h(G_i^B(h) - G_i^B(l)) = (1 - \pi_h)(G_i^B(h) - G_i^B(l))$ for every $i \in I^h$ and $z_i^l(G_i^B(l) - G_i^B(h)) = \pi_l(G_i^B(l) - G_i^B(h))$ for every $i \in I^l$. Note that z_i^s is agent i's willingness to pay to impose realisation s ex post.

Let B be an IC collection and suppose that there is a pair of agents $i \in I^h - M$ and $j \in M$ such that $z_i^h(G_i^B(h) - G_i^B(l)) + G_j^B(h) - G_j^B(l) > c$. Consider a side-contract b_{ji} such that $b_{ji}(l) = \epsilon < b_{ji}(h) = z_i^h(G_i^B(h) - G_i^B(l)) - \epsilon$. Note that if $\epsilon > 0$ is sufficiently small, then $PM(B + b_{ji}) = \{j\}$ and, by Assumption 1, $\sigma(B + b_{ji}, \theta) = h$ for every $\theta \in \Theta$. Moreover, if $\epsilon > 0$ is sufficiently small, both i and j are better off signing this side-contract. By Lemma 2, there is no permissible conjecture $\beta_j(b_{ji})$ that blocks b_{ji} and, therefore, B is not pairwise weakly stable. We can conclude that B is pairwise weakly stable only if there is no pair of agents $i \in I^h - M$ and $j \in M$ such that $z_i^h(G_i^B(h) - G_i^B(l)) + G_j^B(h) - G_j^B(l) > c$. Similarly, a necessary condition for B to be pairwise weakly stable is that there is no pair of agents $i \in I^l - M$ and $j \in M$ such that $z_i^l(G_i^B(l) - G_i^B(h)) + G_j^B(l) - G_j^B(h) > c$.

Step 2. In this step, we solve for the maximal volume of trade that can be attained subject to incentive compatibility, Condition 1, and the two necessary conditions established in the previous paragraph. Since z_i^s depends only on $G_i^B(h) - G_i^B(l)$, there is no loss of generality in focusing on symmetric collections of contracts, namely, collections in which $G_i^B(h) - G_i^B(l) = G_j^B(h) - G_j^B(l)$ for every pair of agents $i, j \in J \in$

$\{I^h - M, I^l - M, I^h \cap M, I^l \cap M\}$. We denote by R^h_k, R^l_k, R^h_m, R^l_m the exposure to the signal $G^B_i(h) - G^B_i(l)$ of each $i \in I^h - M$, $i \in I^l - M$, $i \in I^h \cap M$, and $i \in I^l \cap M$, respectively. Thus, we can translate the necessary conditions identified in Step 1 to

$$R^h_m + z^h(R^h_k) \leq c \tag{12}$$

$$-R^l_m + z^l(R^l_k) \leq c. \tag{13}$$

The incentive-compatibility constraints together with Condition 1 imply that

$$R^l_k \leq 0, \quad R^h_k \geq 0, R^l_m \in [-c, 0], \quad and \quad R^h_m \in [0, c]. \tag{14}$$

Since contracts are budget-balanced, it holds that

$$\alpha R^h_m + (1-\alpha)R^h_k = -(\alpha R^l_m + (1-\alpha)R^l_k). \tag{15}$$

To solve for the maximal average volume of trade, we can maximise $\alpha R^h_m + (1-\alpha)R^h_k$ subject to (12) and (14), and maximise $-\alpha R^l_m - (1-\alpha)R^l_k$ subject to (13) and (14). The minimum of the two is the maximal average coverage and it is given in the premise of the proposition.

Step 3. We now construct a collection of contracts that induces a volume of trade as in (1) while being IC, IR, pairwise weakly stable, and satisfying Condition 1. There are two cases to consider: (1) $\alpha \geq (1-\alpha)min\{\frac{1}{1-\pi_h}, \frac{1}{\pi_l}\}$ and (2) $\alpha < (1-\alpha)min\{\frac{1}{1-\pi_h}, \frac{1}{\pi_l}\}$.

Case 1. Let $I^h \cap M = \{1, ..., N\}$ and $I^l \cap M = \{1', ..., N'\}$. For each $i \in I^h \cap M$ match agent $i' \in I^l \cap M$ and set a contract $b_{ii'}$ such that $b_{ii'}(h) - b_{ii'}(l) = c$, $\pi_h b_{ii'}(h) + (1-\pi_h)b_{ii'}(l) > 0$, and $-\pi_l b_{ii'}(h) - (1-\pi_l)b_{ii'}(l) > 0$. Since $\pi_h > \pi_l$, such contracts exist. Denote the collection of these contracts by B'. In addition to these contracts, for each pair of agents $i, j \in I^h \cap M$ set two contracts:

- $b^1_{ij}(l) = -10c$
- $b^1_{ij}(h) = z^h_i(c)$
- $b^2_{ij}(l) = 10c$
- $b^2_{ij}(l) = -z^h_i(c)$

and for each pair of agents $i, j \in I^l \cap M$ set two contracts:

- $b^1_{ij}(h) = -10c$
- $b^1_{ij}(l) = z^l_i(c)$
- $b^2_{ij}(h) = 10c$
- $b^2_{ij}(l) = -z^l_i(c)$.

Denote the collection of all contracts by B and note that, by construction, B is IR (no agent wants to cancel the single contract that he signed in B' and the transfers each agent obtains in the contracts $B - B'$ sum up to zero) and IC. It also satisfies Condition 1. We now verify that no agent is better off cancelling a contract $b \in B - B'$.

Suppose that agent $i \in I^h \cap M$ cancels b^1_{ij}. Assumption 2 implies that $PM(B - b^1_{ij}) = \{i, j\}$ and that $\sigma(B - b^1_{ij}, \theta) = h$ for every $\theta \in \Theta$. Moreover, agent i obtains transfers of $G^B_i(h) - z^h_i(c)$ regardless of the state under $B - b^1_{ij}$. Thus, i is indifferent whether to cancel b^1_{ij} or not. Agent j is strictly worse off cancelling b^1_{ij} as he obtains a transfer of $G^B_i(h) + z^h_i(c)$ in this case and incurs a cost of $10c + z^h_i(c) - c$. The analysis of the cancellation of every other contract in $B - B'$ is either identical to the analysis of the cancellation of b^1_{ij} or a mirror image of it. Therefore, we omit it. We can conclude that no agent is strictly better off cancelling a contract in B unilaterally.

It is left to check that for every side-contract b_{ij} such that $B + b_{ij} \succ_i B$, there exists a permissible conjecture $\beta_j(b_{ij})$ that blocks it.

First, consider an agent $i \in I^h - M$ and a side-contract b_{ij} such that $B + b_{ij} \succ_i B$. If $j \in I^h \cup (I^l \cap M)$, then $B + b_{ij} \succ_i B$ implies that $B \succ_j B + b_{ij}$. Thus, the conjecture $\beta_j(b_{ij}) = \emptyset$ is permissible and blocks b_{ij}. Suppose that $j \in I^l - M$. In this case, $B + b_{ij} \succ_i B$ together with $B \not\succ_j B + b_{ij}$ implies that $b_{ij}(h) > 0 > b_{ij}(l)$. The side-contract b_{ij} is blocked by a conjecture $\beta_j(b_{ij}) = b_{xi}$, where $x \in I^h \cap M$, $b_{xi}(h) = 2\epsilon > \epsilon = b_{xi}(l)$, and ϵ is close to 0. This conjecture is consistent since under it the side-contract b_{ij} is a positive transfer from j to i. By Lemma 2, the conjecture is not blocked by any consistent conjecture and, therefore, it is permissible.

Second, consider an agent $i \in I^h \cap M$ and a side-contract b_{ij} such that $B + b_{ij} \succ_i B$. If $j \in I^l \cup (I^h - M)$, then $B + b_{ij} \succ_i B$ implies that $B \succ_j B + b_{ij}$. Thus, the conjecture $\beta_j(b_{ij}) = \emptyset$, which is permissible, blocks b_{ij}. Suppose that $j \in I^h \cap M$. The analysis of this case is similar to the analysis of cases 3 and 4 in Proposition 6 (replacing CARA with risk neutrality) and therefore omitted.

The analysis of the cases where $i \in I^l - M$ and $i \in I^l \cap M$ is analogous to that of the above two cases and omitted for brevity.

Case 2. Assume without loss of generality that $\frac{1}{1-\pi_h} \leq \frac{1}{\pi_l}$. Let $I^h \cap M = \{1, ..., N\}$ and $I^l \cap M = \{1', ..., N'\}$. For every $i \in I^h \cap M$ set a contract with $i' \in I^l \cap M$ such that $(b_{ii'}(h) - b_{ii'}(l))(1 - \pi_h) = c$, $\pi_h b_{ii'}(h) + (1 - \pi_h) b_{ii'}(l) > 0$, and $-\pi_l b_{ii'}(h) - (1 - \pi_l) b_{ii'}(l) > 0$. Since $\pi_h > \pi_l$, such contracts exist. Denote the collection of these contracts by B'' and note that no agent wants to cancel the single contract that he signed in B'' and that B'' is IC and satisfies Condition 1. In addition, define q_i, z_i, and the collection B' as in Step 3 of the proof of Proposition 6 (where q_i and z_i are defined with respect to $G^{B''}$ instead of with respect to $\hat{g}$). As in Proposition 6, the transfers in the contracts in B' sum up to zero and no agent strictly benefits from unilaterally cancelling a contract in $B = B' + B''$.

It is left to check that for every side-contract b_{ij} such that $B + b_{ij} \succ_i B$, there exists a permissible conjecture $\beta_j(b_{ij})$ that blocks it. Let $i \in I^l - M$ and consider a side-contract b_{ij} such that $B + b_{ij} \succ_i B$. If $PM(B + b_{ij}) = \emptyset$, then $B + b_{ij} \succ_i B$ and $B + b_{ij} \not\succ_j B$ imply that $b_{ij}(h) < 0 < b_{ij}(l)$. As in Case 1 of Proposition 6 a conjecture $\beta_j(b_{ij}) = -g^{i,j,k,z}$ is permissible and blocks the deviation (under such a conjecture, b_{ij} becomes a state-independent transfer of $b_{ij}(l) > 0$ from j to i).

Now suppose that $PM(B+b_{ij}) \neq \emptyset$. It follows that $PM(B+b_{ij}) = \{j\}$. Moreover, $B+b_{ij} \succ_i B$ implies that $\sigma(B+b_{ij}, \theta) = l$ for every $\theta \in \Theta$ and that $-b_{ij}(l) < z^l(R^l_k) \leq c$. Since $j \in PM(B + b_{ij})$, (13) implies that $b_{ij}(h) > 0$ (as, otherwise, j's incentive constraint is not violated). To see that a conjecture $\beta_j(b_{ij}) = -\overline{g}^{i,j',k',z'}$ is consistent, note that either $\sigma(B+b_{ij}-\overline{g}^{i,j',k',z'}, \theta) = l$ for every $\theta \in \theta$ or $\sigma(B+b_{ij}-\overline{g}^{i,j',k',z'}, \theta) = h$ for every $\theta \in \Theta$. In the former case, $B - \overline{g}^{i,j',k',z'} + b_{ij} \succ_i B + b_{ij} \succ_i B \sim_i B - \overline{g}^{i,j',k',z'}$. In the latter case, b_{ij} is a state-independent transfer of $b_{ij}(h) > 0$ from j to i. Since in both cases j is worse off signing b_{ij} (taking the cost of setting the signal into account) we have a permissible conjecture that blocks the deviation.

Now consider the case where $i \in I^l \cap M$. If $PM(B + b_{ij}) = \emptyset$, then $B + b_{ij} \succ_i B$ and $B \not\succ_j B + b_{ij}$ imply that $b_{ij}(l) > 0 > b_{ij}(h)$ and $|b_{ij}(l) - b_{ij}(h)| \leq c$. Thus, under a conjecture $\beta_j(b_{ij}) = -g^{i,j',k',z'}$, b_{ij} is essentially a state-independent transfer of $b_{ij}(l) > 0$ from j to i. Hence, the conjecture is consistent and blocks the deviation. It is also permissible as it does not require the consent of a third party.

Now suppose that $PM(B+b_{ij}) \neq \emptyset$. If $PM(B+b_{ij}) = \{i, j\}$, then $\sigma(B+b_{ij}, \theta) = h$ for every $\theta \in \Theta$ and $B + b_{ij} \succ_i B$ implies that $B \succ_j B + b_{ij}$. Thus, the permissible conjecture $\beta_j(b_{ij}) = \emptyset$ blocks b_{ij}. Note that it cannot be that $PM(B + b_{ij}) = \{j\}$. To complete the proof, assume that $PB(B + b_{ij}) = \{i\}$. The analysis of the case where $PM(B + b_{ij}) = \{i\}$ is similar to that of Case 2 in the proof of Proposition 6 and therefore it is omitted.

The analysis of the case where $i \in I^l - M$ and the case where $i \in I^l \cap M$ is analogous to that of the above two cases and omitted for brevity.

Supplementary Appendix

The Maximal Aggregate Coverage in Section 5.1

First, let us plug the weak-stability constraint given in (8) into our objective function, $\alpha R_m + (1-\alpha) R_k$, to obtain (16):

$$\alpha \left(c - \frac{1}{\gamma} log[\frac{\pi exp[\gamma R_k] + (1-\pi) exp[\gamma w]}{\pi + (1-\pi) exp[\gamma w]}] \right) + (1-\alpha) R_k. \tag{16}$$

We now maximise (16) subject to the restriction to $R_k \in [0, w]$ and $R_m \in [0, c]$. From the first-order condition we obtain that in an internal solution,

$$R_k = w - \frac{1}{\gamma} log[\frac{\pi (2\alpha - 1)}{(1-\pi)(1-\alpha)}]. \tag{17}$$

The coverage R_k is decreasing in π since the willingness to pay to guarantee that the signal is l is increasing in the probability that state H is realised. This follows from the fact that $k \in L - M$ benefits from manipulation only when state H is realised. Intuitively, when α is increasing, R_k is decreasing and R_m is increasing. Moreover, as explained in the main text, the coverage R_k increases when agents become more risk-averse. In an internal solution,

$$R_m = \left(c - \frac{1}{\gamma} log[\frac{\pi exp[\gamma \left(w - \frac{1}{\gamma} log[\frac{\pi(2\alpha-1)}{(1-\pi)(1-\alpha)}] \right)] + (1-\pi) exp[\gamma w]}{\pi + (1-\pi) exp[\gamma w]}] \right). \tag{18}$$

Hence the maximal coverage per insurer that can be obtained in an IR, IC, and pairwise weakly stable contract is

$$\alpha \left(c - \frac{1}{\gamma} log[\frac{\pi exp[\gamma (w - \Delta)] + (1-\pi) exp[\gamma w]}{\pi + (1-\pi) exp[\gamma w]}] \right) + (1-\alpha)(w - \Delta)], \tag{19}$$

where $\Delta := \frac{1}{\gamma} log[\frac{\pi(2\alpha-1)}{(1-\pi)(1-\alpha)}]$. To ensure that the solution is indeed internal[10] we must verify that $R_k \in (0, w)$. Observe that this condition is satisfied for $\alpha \in (\frac{1}{1+\pi}, \frac{\pi+(1-\pi)exp[\gamma w]}{2\pi+(1-\pi)exp[\gamma w]})$.

Let us consider the following parameters: $w = 10, \gamma = 0.5, \pi = 0.9$, and $c = 8$. For $\alpha \in (0.526, 0.945)$ we obtain an internal solution for R_k. For $\alpha \leq 0.526$ ($\alpha \geq 0.945$),

[10]In a corner solution, either $R_k = w$ or $R_k = 0$ (plugging R_k into (16) yields the maximal average coverage).

$R_k = 10$ ($R_k = 0$) and $R_m = 3.512$ ($R_m = 8$). For $\alpha = 0.75$, the maximal average level of coverage is 6.534, which reflects a loss of 23.13 percent when compared to the constrained-efficient level of coverage, which is 8.5. If we increase the agents' risk aversion to $\gamma = 1$, we have an internal solution to R_k for $\alpha \in (0.526, 0.999)$. For $\alpha \leq 0.526$ ($\alpha \geq 0.999$), we have $R_k = 10$ ($R_k = 0$) and $R_m = 5.697$ ($R_m = 8$).

Mixed Conjectures

In this part of the appendix, we incorporate mixed conjectures into the analysis and show that allowing for mixed conjectures does not change any of the results in the paper. To this end, we shall prove that a collection of contracts is weakly stable when mixed conjectures are allowed if and only if it is weakly stable when mixed conjectures are precluded.

Incorporating mixed conjectures into the model requires adapting our basic notions. For every collection of contracts $B \in \mathcal{B}$ and pair of agents $i, j \in I$, let $D^0_{ij}(B) = \{g^Z \in B | i \in Z \textit{ and } j \notin Z\}$ be the set of contracts in B that involve i but not j, and let $D^1_{ij}(B) = \{g^Z \notin B | i \in Z \textit{ and } j \notin Z\}$ be the set of potential side-contracts that involve agent i and not j. An element of $D^0_{ij}(B)$ represents a cancellation of a contract by i and an element of D^1_{ij} represents a signing of a side-contract by i. A mixed conjecture $\hat{\beta}_j(B, g^K, i) \in \Delta(D^0_{ij}(B) \cup D^1_{ij}(B))$ is a probability distribution over deviations that involve agent i. We reserve β to denote "pure" conjectures and, as in the main text, omit B from the description of a conjecture. Recall that a pure conjecture $\beta_j(g^K, i)$ is consistent if $B + \beta_j(g^K, i) + g^K \succ_i B + \beta_j(g^K, i)$. A mixed conjecture $\hat{\beta}_j(g^K, i)$ is said to be consistent if every pure conjecture $\beta_j(g^K, i) \in supp(\hat{\beta}_j(g^K, i))$ is consistent. A conjecture $\hat{\beta}_k(g^Z, i)$ blocks the conjecture $\hat{\beta}_j(g^K, i)$ if $g^Z \in supp(\hat{\beta}_j(g^K, i))$, $g^Z \in D^1_{ij}(B)$, $k \in Z - \{i\}$, and $B + \hat{\beta}_k(g^Z, i) \succ_k B + \hat{\beta}_k(g^Z, i) + g^Z$.

We are now ready to incorporate mixed conjectures into the notion of permissibility.

Definition 9 *For every collection $B \in \mathcal{B}$, side-contract g^K, agent $j \in K$, and initiator $i \in K - \{j\}$, let*

$$\hat{A}^0_j(B, g^K, i) = \{\hat{\beta}_j(g^K, i) \in \Delta(D^0_{ij}(B)) | \hat{\beta}_j(g^K, i) \textit{ is consistent}\},$$

and, for every $t > 0$, let $\hat{A}^t_j(B, g^K, i)$ be the set of consistent conjectures $\hat{\beta}_j(g^K, i)$ that satisfy the following condition:

- *For every consistent conjecture $\hat{\beta}_k(g^Z, i)$ that blocks $\hat{\beta}_j(g^K, i)$, there exists a conjecture $\hat{\beta}_z(g^J, i) \in \cup_{x=0}^{t-1} \hat{A}_z^x(B, g^J, i)$ that blocks it.*

A conjecture $\hat{\beta}_j(g^K, i)$ is said to be permissible if $\hat{\beta}_j(g^K, i) \in \cup_{t=0}^{\infty} \hat{A}_j^t(B, g^K, i)$.

The definition of weak stability remains as in the main text.

Definition 10 *A collection of contracts B is said to be weakly stable if the following two conditions are met:*

- *There is no agent $i \in I$ and contract $g^K \in B$ such that $i \in K$ and $B - g^K \succ_i B$.*
- *For every side-contract g^K such that $B + g^K \succ_i B$ for some $i \in K$, there exists an agent $j \in K - \{i\}$ and a permissible conjecture $\hat{\beta}_j\left(g^K, i\right)$ that blocks g^K.*

Proof of the equivalence result.

First, observe that a side-contract g^K is blocked by a mixed conjecture $\hat{\beta}_j(g^K, i)$ only if it is blocked by at least one pure conjecture $\beta_j(g^K, i) \in supp(\hat{\beta}_j(g^K, i))$. This implies that a side-contract is blocked by a mixed conjecture if and only if it is blocked by a pure conjecture.

The second step of the proof is to show that the set of pure permissible conjectures is the same regardless of whether mixed conjectures are allowed or not. To see this, consider an arbitrary side-contract g^K and the set of pure conjectures $A_j^0(g^K, i)$ (we omit "B" for brevity). Every pure conjecture $\beta_j(g^K, i) \in A_j^0(g^K, i)$ is consistent and $\beta_j(g^K, i) \in D_{ij}^0(B)$. Hence, $A_j^0(g^K, i) \subseteq \hat{A}_j^0(g^K, i)$. Note that $\hat{A}_j^0(g^K, i) - A_j^0(g^K, i)$ consists only of mixed conjectures whose support consists of pure consistent conjectures that i cancelled a contract. Thus, it includes only conjectures whose support consists of pure conjectures in $A_j^0(g^K, i)$.

We now focus on $\hat{A}_j^1(g^K, i)$. There are three things we need to show. First, $(A_j^0(g^K, i) \cup A_j^1(g^K, i)) \subseteq (\hat{A}_j^0(g^K, i) \cup \hat{A}_j^1(g^K, i))$. Second, $\hat{A}_j^0(g^K, i) + \hat{A}_j^1(g^K, i) - A_j^0(g^K, i) - A_j^1(g^K, i)$ does not include pure conjectures. Third, every pure conjecture in the support of a mixed conjecture $\hat{\beta}_j(g^K, i) \in \hat{A}_j^1(g^K, i)$ must belong to $A_j^0(g^K, i) \cup A_j^1(g^K, i)$. It is then possible to show that these three properties hold for any $t > 1$ by induction.

Consider a pure conjecture $\beta_j(g^K, i) = g^Z$ such that $g^Z \in A_j^1(g^K, i)$. It is consistent regardless of whether mixed conjectures are allowed. Moreover, by definition, there is no conjecture $\beta \in \cup_{k\in Z-\{i\}} A^0_k(g^Z, i)$ that blocks it. Since every conjecture in $\hat{A}^0_k(g^Z, i)$ is a mix of pure conjectures that belong to $A^0_k(g^Z, i)$, the first step of the proof shows that $\beta_j(g^K, i)$ is not blocked by any conjecture $\beta \in \cup_{k\in Z-\{i\}} \hat{A}^0_k(g^Z, i)$. Finally, note that every pure consistent conjecture $\beta_k(g^Z, i) = g^J$ that blocks $\beta_j(g^K, i) = g^Z$ is itself blocked by a conjecture $\beta \in \cup_{z\in J-\{i\}} A^0_z(g^J, i) \subseteq \cup_{z\in J-\{i\}} \hat{A}^0_z(g^J, i)$. We can conclude that $\beta_j(g^K, i) \in \hat{A}^0_j(g^K, i) \cup \hat{A}^1_j(g^K, i)$. Hence, $(A^0_j(g^K, i) \cup A^1_j(g^K, i)) \subseteq (\hat{A}^0_j(g^K, i) \cup \hat{A}^1_j(g^K, i))$.

Consider a pure conjecture $\beta_j(g^K, i) = g^Z$ such that $g^Z \notin A^1_j(g^K, i) \cup A^0_j(g^K, i)$. Either it is not consistent or it is consistent and there exists some agent $k \in Z - \{i\}$ and a consistent conjecture $\beta_k(g^Z, i) = g^J$ that blocks $\beta_j(g^K, i) = g^Z$ and is not blocked by any pure conjecture $\beta \in \cup_{z\in J-\{i\}} A^0_z(g^J, i)$. In either case, it cannot be part of $\hat{A}^1_j(g^K, i)$. We can conclude that $\hat{A}^0_j(g^K, i) + \hat{A}^1_j(g^K, i) - A^0_j(g^K, i) - A^1_j(g^K, i)$ does not include pure conjectures.

Finally, assume to the contrary that there is a mixed conjecture $\hat{\beta}_j(g^K, i) \in \hat{A}^1_j(g^K, i)$ and a pure conjecture $\beta_j(g^K, i) \in supp(\hat{\beta}_j(g^K, i))$ such that $\beta_j(g^K, i) \notin A^1_j(g^K, i) \cup A^0_j(g^K, i)$. If $\beta_j(g^K, i)$ is inconsistent, then we obtain a contradiction to the consistency of $\hat{\beta}_j(g^K, i)$. Suppose that $\beta_j(g^K, i)$ is consistent and let $\beta_j(g^K, i) = g^Z$. Since $g^Z \notin A^1_j(g^K, i) \cup A^0_j(g^K, i)$, it must be blocked by a consistent conjecture $\beta_k(g^Z, i)$ that is itself not blocked by any consistent conjecture $\beta \in \cup_{z\in\beta_k^{-1}(g^Z,i)} A^0_z(\beta_k(g^Z, i), i)$. This is in contradiction to the fact that both $\hat{\beta}_j(g^K, i) \in \hat{A}^1_j(g^K, i)$ and $\beta_j(g^K, i) \in supp(\hat{\beta}_j(g^K, i))$.

We conclude that the set of pure permissible conjectures is unchanged when mixed conjectures are allowed. Recall that a side-contract is blocked by a mixed conjecture only if it is blocked by one of the pure conjectures in its support. Hence, an IC collection of contracts is weakly stable when mixed conjectures are allowed if and only if it is weakly stable when mixed conjectures are not allowed.