Searching Forever After

By Yair Antler and Benjamin Bachi*

We study a model of two-sided search in which agents' strategic reasoning is coarse. In equilibrium, the most desirable agents behave as if they were fully rational, while, for all other agents, coarse reasoning results in overoptimism with regard to their prospects in the market. Consequently, they search longer than optimal. Moreover, agents with intermediate match values may search indefinitely while all other agents eventually marry. We show that the share of eternal singles converges monotonically to 1 *as search frictions vanish. Thus, improvements in search technology may backfire and even lead to market failure.*

Modern search technologies present new opportunities for individuals who are looking for a partner. For instance, mobile applications such as Tinder and Bumble, and online dating sites such as OkCupid and Plenty of Fish, allow individuals to find a partner with the swipe of a finger. These new technologies have reduced search costs and thickened matching markets, which enables individuals to meet a large number of potential matches in a short span of time.

Choosing a partner is one of the most important decisions in a person's life. It typically entails comparing a specific potential partner to a risky outside option, that is, continuing to search without knowing for how long or with whom one will eventually partner. Assessing this outside option requires understanding other people's behavior, which can be challenging. It may lead individuals to use heuristics and simplified models of the world to assess their prospects, especially in light of the wide array of options that are ubiquitous in modern matching markets.

This paper studies how advances in search technology affect marriage market outcomes when agents' reasoning is imperfect. To this end, we study a model of two-sided search with vertical differentiation and nontransferable utility. This framework has proved useful in understanding decentralized matching markets (see Chade, Eeckhout and Smith, 2017, for a comprehensive review). In this framework, agents are matched at random and decide whether to accept the match or continue to search. It is typically assumed that the participants are fully rational and, in particular, can perfectly assess the prospect of remaining

* Antler: Coller School of Management, Tel Aviv University, Tel Aviv, Israel, 69978 (email: yair.an@gmail.com); Bachi: Department of Economics, University of Haifa, Haifa, 3498838, Israel (email: bbachi@econ.haifa.ac.il). We thank Daniel Bird, Kfir Eliaz, Tobias Gamp, Yuval Heller, Philippe Jehiel, Johannes Johnen, Dotan Persitz, Santiago Oliveros, Ariel Rubinstein, Yossi Spiegel, Rani Spiegler, and four anonymous referees for helpful comments and suggestions. We also thank seminar and conference audiences at Bar Ilan University, EEA 2020, ESWC 2020, HU Berlin, Tel Aviv University, University of East Anglia BGT Workshop, and University of Essex for useful comments. Antler gratefully acknowledges financial support from the Henry Crown Institute of Business Research in Israel.

single and continuing to search. Under that assumption, improvements in the search technology enhance the welfare of the bulk of the market participants. However, in practice, not everyone benefits from these changes. For example, according to a survey by Pew Research Center (2016), "One-third of people who have used online dating have never actually gone on a date with someone they met on these sites."

We depart from the rational expectations paradigm by relaxing the assumption that agents have a perfect understanding of the mapping from the other agents' characteristics to their behavior. The literature offers two main equilibrium notions that capture the above idea: the partially cursed equilibrium (Eyster and Rabin, 2005) and the analogy-based expectation equilibrium (Jehiel, 2005). We characterize the equilibria of the model under both notions and find that, despite their differences, they lead to similar results.

In equilibrium, except for the most desirable agents, who behave as if they were fully rational, all other agents are overoptimistic with regard to the prospect of remaining single and continuing to search. This overoptimism has significant implications as, except for the least desirable agents who accept all potential partners, agents who overvalue the prospect of remaining single reject agents whom a rational agent would accept. In a two-sided market, this imposes a negative externality on agents on the other side of the market and causes them to search longer as fewer people are willing to marry them. In equilibrium, these effects lead to a delay in matching. In fact, as long as the meeting rate is not too slow, in every symmetric equilibrium, there are agents with intermediate match values (i.e., moderately desirable agents) who search indefinitely and remain single forever. By contrast, agents with lower or higher match values marry in finite time.

We show that when search frictions become less intense, the share of agents who search indefinitely weakly increases. Thus, technological improvements that result in faster search that enhance individuals' welfare if they are fully rational can degrade it if they are not. Though counterintuitive, the finding that fewer matches are formed when the market becomes less frictional is consistent with recent empirical findings: Fong (2020) shows that when more men and women join a dating platform, participants in the market become more selective and the number of matches per individual goes down.

We find that, for any level of coarseness (or, in Eyster and Rabin's terminology, any degree of partial cursedness), the share of agents who search indefinitely converges to 1 when search frictions vanish. The intuition for the market collapse is as follows. Our agents falsely believe that "top" agents are achievable. When the technology improves and allows potential partners to meet more frequently, their willingness to wait for a top agent increases and they become more selective. Eventually, agents become too selective and reject agents of their own caliber or lower. For similar reasons, they are rejected by agents of their own caliber or higher. As a result, they search indefinitely and never marry.

These market unraveling results are quite different from the results under the rational expectations model, in which, when frictions vanish, the equilibrium converges to a stable matching in the sense of Gale and Shapley (1962). This contrast highlights that even a slight departure from the rational expectations model can lead to radically different outcomes when search frictions vanish.

Related Literature

Our paper contributes to a large body of literature on matching with frictions (see McNamara and Collins, 1990; Burdett and Coles, 1997; Eeckhout, 1999; Bloch and Ryder, 2000; Shimer and Smith, 2000; Chade, 2001, 2006; Adachi, 2003; Smith, 2006). This literature focuses on the properties of induced matchings under various assumptions on search frictions, match payoffs, search costs, and the ability to transfer utility. In particular, it shows that when utility is nontransferable and frictions vanish, equilibrium matching converges to an efficient matching.[1]

We adopt the coarse reasoning models of Eyster and Rabin (2005) and Jehiel (2005). Similar ideas have been applied in various contexts. For example, Piccione and Rubinstein (2003) study intertemporal pricing, where consumers think in terms of a coarse representation of the equilibrium price distribution.[2] In the context of consumer search, Gamp and Krähmer (2018) analyze a model in which a share of consumers do not distinguish between deceptive and candid products nor can they infer quality from price. In Gamp and Krähmer (2019), a share of the consumers misestimate the correlation between price and quality and, as a result, search excessively for a high-quality low-priced product, falsely believing that it exists. These false expectations stimulate competition between fully rational sellers and the effect is most intense when the consumers' search costs (or level of misestimation) are intermediate.[3]

In the two-sided search framework, coarse reasoning leads to selection neglect in equilibrium. Esponda (2008) proposes an equilibrium model of selection neglect and shows that traders who do not account for selection can exacerbate adverse selection problems. In Jehiel (2018), entrepreneurs decide whether or not to invest in a project based on feedback from implemented projects. The entrepreneurs ignore the lack of feedback from unimplemented projects, which, on average, are inferior to implemented ones. As a result they become overoptimistic and implement projects in cases where it is suboptimal to do so.

There are a limited number of theoretical papers that relax the full rationality assumption in the context of matching. Eliaz and Spiegler (2014) analyze a

[1]Lauermann and Nöldeke (2014) find conditions under which this result holds without vertical heterogeneity.

[2]Other applications are Jehiel (2011) in the context of auctions, Eyster and Piccione (2013), Steiner and Stewart (2015), Kondor and Kőszegi (2017), and Eyster, Rabin and Vayanos (2019) in the context of trade in financial markets, and Antler (2018) in the context of pyramid schemes.

[3]In these models, agents can be viewed as if they were using a simplified representation of the world to form their expectations. For a comprehensive review of equilibrium models in which individuals interpret data by means of a misspecified causal model see Spiegler (2020).

search-and-matching model where agents exhibit "morale hazard." The behavioral assumption in that paper pertains to the agents' preferences rather than to their beliefs. In the context of centralized matching, a recent strand of the literature assumes that agents' preferences are nonstandard (see, e.g., Antler, 2015; Fernandez, 2018; Dreyfuss, Heffetz and Rabin, 2019; Meisner and von Wangenheim, 2019).

Our departure from the rational expectations setting is in line with empirical evidence that people neglect correlations when problems become more complex (Enke and Zimmermann, 2019). In the context of centralized school matching, Rees-Jones, Shorrer and Tergiman (2020) find that students tend to neglect correlation between schools' tastes and priorities. In the context of courtship, Fisman et al. (2006) find that men exhibit behavior consistent with choice overload, and Lenton and Francesconi (2011) document similar findings on both sides of the market.

The paper proceeds as follows. Section I presents the baseline model and benchmark results in a setting in which agents who leave the market are replaced by identical ones. Section II studies the behavioral models in this setting. Section III analyzes the steady-state equilibrium when entry into the market is exogenous. Section IV concludes. All proofs are relegated to the Appendix.

I. The Baseline Model

There is a set of men, $\mathcal{M}$, and a set of women, $\mathcal{W}$, each containing a unit mass of agents. Each agent is characterized by a number, which, following Burdett and Coles (1997), we refer to as the agent's pizzazz and assume is distributed on the interval $[\underline{v}, \overline{v}]$, $\underline{v} > 0$, according to an atomless continuous distribution F. We denote the corresponding density by f and refer to an agent with pizzazz v as agent v.

The market operates in continuous time. Each individual meets agents of the opposite sex at a flow rate μ, where μ is the parameter of a Poisson process. Meetings are random: agents meet agents of the opposite sex with pizzazz value in $[v_1, v_2]$ at a flow rate proportional to their mass in the population $\mu[F(v_2)-F(v_1)]$. When two agents meet, they immediately observe each other's pizzazz and decide whether to accept each other as a partner. If both agents accept, then they marry and exit the market. Otherwise, they return to the market and continue their search. When agent v marries agent w, the latter obtains a payoff of v and the former obtains a payoff of w. Agents obtain no flow payoff when single. Agents maximize their expected payoffs discounted at a rate $r > 0$.

When agents leave the market they are immediately replaced by agents with identical characteristics and so the distribution of agents' characteristics does not change over time. This simplifying assumption allows us to focus on the main messages while keeping the exposition simple. As we show in Section III, when considering a richer model with exogenous inflow in which married agents are not replaced by clones, there indeed exists a steady-state equilibrium in which the distribution of agents' characteristics does not change over time.

A stationary strategy for agent v, $\sigma_v(\cdot) : [\underline{v}, \overline{v}] \to \{1, 0\}$, is a mapping from pizzazz values of agents on the other side of the market to a decision whether to accept or reject a match.[4] Throughout the analysis, we assume that agents use cutoff strategies: each agent v's strategy is characterized by a cutoff $\hat{a}_v$ such that agent v accepts matches with agents whose pizzazz is at least $\hat{a}_v$ and rejects all others.[5] For each agent v and profile of strategies σ, let $A_v(\sigma) = \{w|\sigma_w(v) = 1\}$ be the set of agents who accept a match with v and let $a_v(\sigma) = \sup(A_v(\sigma))$ be agent v's opportunity value. When there is no risk of confusion, we omit the dependence on σ from A_v and a_v.

Throughout the analysis, we focus on symmetric equilibria, namely, equilibria in which women and men with the same pizzazz use the same strategy. We discuss and illustrate the differences between symmetric and asymmetric equilibria at the end of Section II.A, after presenting our results.

Benchmark Results: Full Rationality

The analysis of the "rational expectations" benchmark follows from well-known results in the matching with frictions literature and, therefore, we omit the formal proofs. Proposition 1 is a classic block segregation result (see, e.g., McNamara and Collins, 1990; Burdett and Coles, 1997; Eeckhout, 1999; Bloch and Ryder, 2000; Chade, 2001; Smith, 2006).

PROPOSITION 1: *There exist numbers $\overline{v} = v^0 > v^1 > v^2 > ... > v^N = \underline{v}$ such that, in the unique equilibrium, every agent $v \in [v^{j+1}, v^j)$ uses the acceptance cutoff v^{j+1}.*

In equilibrium, agents are partitioned into *classes*, such that agents who belong to the same class use the same acceptance cutoff and have the same opportunity value. All agents are accepted by members of their class and rejected by members of higher classes. Similarly, all agents find members of their class to be acceptable and reject members of lower classes. Thus, agents marry within their class in finite time.

By Proposition 1, in equilibrium, the agents' pizzazz is strictly greater than their acceptance cutoffs, except for agents at the lower bound of a class. When search frictions vanish (e.g., when μ becomes infinitely large), the induced matching converges to the unique stable matching (Eeckhout, 1999; Bloch and Ryder, 2000; Adachi, 2003), which implies that married couples have the same pizzazz. Thus, when search frictions vanish, the classes shrink and almost all of the agents'

[4] Abusing notation, we can think of σ_v as a subset of agents on the other side of the market whom v accepts. Then, $\sigma : [\underline{v}, \overline{v}] \to \mathcal{P}([\underline{v}, \overline{v}])$ is a correspondence, which we assume to be measurable.

[5] Assuming that agents' use cutoff strategies is equivalent to assuming that they break indifference in favor of accepting a match. The results in this paper are not sensitive to this assumption. In particular, the results hold if indifferent agents accept matches with probability $q > 0$.

acceptance cutoffs increase. These increases can be interpreted as an increase in the agents' welfare.

II. Coarse Reasoning in the Matching Market

Consider an agent who faces a decision whether to accept or reject a potential partner. Since this decision has implications only when the potential partner accepts, the agent essentially compares the known payoff from marrying the partner to the risky option of remaining single and continuing to search. Assessing the latter option requires some understanding of the future behavior of agents of the opposite sex.

Agents in our model have a coarse perception of the future behavior of agents on the other side of the market. They understand the rate at which they are accepted by potential partners, but do not discern exactly who finds them acceptable and who does not. The two most prominent approaches that capture this idea are the partially cursed equilibrium (Eyster and Rabin, 2005) and the analogy-based expectation equilibrium (Jehiel, 2005). In Section II.A, we take the first approach, which was originally developed for Bayesian games, and adapt it to our setting. This equilibrium concept captures the idea that agents make mistakes with respect to the whole population and allows us to vary the size of the mistake. In Section II.B, we take the second approach and apply it to our model. This equilibrium concept captures the idea that agents make mistakes with respect to only a fraction of the population and allows us to vary the size of this fraction.

A. *The Partially Cursed Equilibrium*

Cursed agents have an imperfect understanding of the mapping from the other agents' pizzazz to their matching decisions. *Fully cursed* agents believe that every agent of the opposite sex accepts them as a partner with a probability that equals the average probability at which they are accepted by the entire population on the other side of the market. *Partially cursed* agents understand that the other agents' behavior depends on their pizzazz but do not understand to what extent. Specifically, a partially cursed agent v believes that an agent w on the other side of the market will accept her/him as a partner with a probability that is a convex combination of the true probability with which w accepts v and the average rate at which v is accepted by the entire population.

Given a strategy profile σ, the objective probability that agent w accepts agent v is equal to $\sigma_w(v)$. The average rate at which the entire population accepts agent v is

$$\int_{\underline{v}}^{\overline{v}} \sigma_x(v) f(x) dx. \tag{1}$$

Thus, a partially cursed agent v believes that agent w will accept her/him as a partner with probability

$$\gamma_v(w) = \psi \int_{\underline{v}}^{\overline{v}} \sigma_x(v) f(x) dx + (1-\psi)\sigma_w(v), \tag{2}$$

where ψ represents the magnitude of the agents' mistakes. Thus, ψ can be thought of as a *behavioral friction.* When $\psi = 0$, agents have rational expectations. At the other extreme, when $\psi = 1$, agents are fully cursed.

As an illustration, suppose that the median pizzazz in the population is w_m and that $A_v = \{w | w < w_m\}$ for some man v. If man v is fully rational, then he expects women whose pizzazz is higher than w_m to accept him with probability 0, and women whose pizzazz is lower than w_m to accept him with probability 1. A partially cursed man v with $\psi = 0.1$ expects women whose pizzazz is higher than w_m to accept him with probability 0.05, and women whose pizzazz is lower than w_m to accept him with probability 0.95. Thus, v overestimates the probability of being accepted by women whose pizzazz is high and underestimates the probability of being accepted by women whose pizzazz is low.[6]

Let $U(\hat{a}_v, \gamma_v)$ be agent v's perceived expected discounted payoff conditional on holding beliefs $\gamma_v(\cdot)$ and using a cutoff $\hat{a}_v$. That is,

$$(1 + rdt)U(\hat{a}_v, \gamma_v) = \mu dt \int_{\hat{a}_v}^{\overline{v}} \gamma_v(x) x f(x) dx + U(\hat{a}_v, \gamma_v)\big(1 - \mu dt \int_{\hat{a}_v}^{\overline{v}} \gamma_v(x) f(x) dx\big).$$

Rearranging and letting $dt \to 0$ yields

$$U(\hat{a}_v, \gamma_v) = \frac{\int_{\hat{a}_v}^{\overline{v}} \gamma_v(x) x f(x) dx}{\eta + \int_{\hat{a}_v}^{\overline{v}} \gamma_v(x) f(x) dx}, \tag{3}$$

where $\eta = r/\mu$ represents the frictions in the market.

DEFINITION 1: *A strategy profile σ forms a partially cursed equilibrium if $\hat{a}_v$ is optimal given γ_v, for each $v \in \mathcal{M} \cup \mathcal{W}$.*

We denote by $U_v^\star$ agent v's perceived expected discounted payoff conditional on using an optimal acceptance cutoff, and refer to it as agent v's *continuation value.* In equilibrium, agent v accepts a match with agent w if and only if $w \geq U_v^\star$, implying that $\hat{a}_v = max\,\{U_v^\star, \underline{v}\}$. The following lemma establishes that, in equilibrium, agents with higher pizzazz have higher standards.

LEMMA 1: *In equilibrium, $\hat{a}_v$ and a_v are weakly increasing in v.*

[6] All the results in the paper hold if the partially cursed equilibrium is modified so that agents' beliefs are correct with respect to agents who accept them and remain partially cursed with respect to agents who reject them.

The assumption that agents use cutoff strategies implies that agents with higher pizzazz are accepted by more agents and, therefore, have better options in the market. This directly implies that a_v is increasing in v. Moreover, since agents with higher pizzazz can always imitate agents with lower pizzazz, it also implies that $\hat{a}_v$ is weakly increasing in v.

The monotonicity of the agents' acceptance cutoffs implies that agent v is accepted by every agent whose pizzazz is lower than a_v and is rejected by every agent whose pizzazz is higher than a_v. Thus, a_v pins down $\gamma_v(\cdot)$. Therefore, in the remainder of the section, we write $U(\hat{a}_v, a_v)$ instead of $U(\hat{a}_v, \gamma_v)$.

Who Marries in Equilibrium. — Under the conventional rational expectations model, if the distribution of agents on both sides of the market is symmetric, then all agents marry in finite time. In our model, the agents' coarse reasoning makes some of them too selective to marry. The next lemma will be useful for understanding who these eternal singles are.

LEMMA 2: *Agent v marries in finite time in a symmetric equilibrium if and only if*

$$U(v, v) < v. \tag{4}$$

Lemma 2 establishes a necessary and sufficient condition for an agent to marry in a symmetric equilibrium. When Condition 4 is violated, an agent with an opportunity value v will prefer remaining single to marrying agent v. Such an agent exhibits a "Groucho Marx" type of behavior, as (s)he is unwilling to marry agents who are willing to marry her/him.

In order to gain intuition for the condition's necessity consider a woman w and a man v. Observe that $U(\hat{a}_w, a_w) \geq U(v, a_w)$ since setting an acceptance cutoff $\hat{a}_w = v$ is not necessarily optimal. If man v accepts woman w in equilibrium, then her opportunity value is at least v, that is, $a_w \geq v$. Since U is increasing in the opportunity value, $U(v, a_w) \geq U(v, v)$. It follows that $U^\star_w = U(\hat{a}_w, a_w) \geq U(v, v)$. Thus, if $U(v, v) > v$, then $U^\star_w > v$, and so every woman w that man v finds acceptable prefers remaining single to marrying him. As a result, v cannot marry in equilibrium.[7]

To gain intuition for the condition's sufficiency, note first that, in a symmetric equilibrium, either $\hat{a}_v \geq v \geq a_v$ or $\hat{a}_v \leq v \leq a_v$. The reason for this is that in a symmetric equilibrium man v accepts woman v if and only if she accepts him. This argument implies that if agent v's opportunity value is strictly greater than v, then (s)he marries in finite time as $\hat{a}_v < a_v$ in this case. Condition 4 implies that the optimal acceptance cutoff of an agent whose opportunity value is v must be strictly lower than v (as the perceived continuation value is lower

[7] The intuition for the necessity of the condition when $U(v, v) = v$ is more subtle and requires utilizing the symmetry assumption.

than the payoff from marrying an agent v). This has two consequences. First, such an agent must marry in finite time. Second, since the optimal acceptance cutoff is increasing in the opportunity value, the optimal acceptance cutoff of an agent whose opportunity value is less than v must also be less than v. However, $a_v < v$ and $\hat{a}_v < v$ cannot hold together in a symmetric equilibrium. Thus, when Condition 4 holds, in a symmetric equilibrium, $a_v \geq v \geq \hat{a}_v$ with at least one strict inequality, and so agent v marries in finite time.

Observe that $U(v, v)$ depends only on the primitives of the model. To see this, note that if $a_w = v = \hat{a}_w$, then $\gamma_w(x) = \psi F(v)$ for any $x > \hat{a}_w$. Thus,

$$(5) \qquad U(v,v) = \frac{\int_v^{\overline{v}} \psi F(v) x f(x) dx}{\eta + \int_v^{\overline{v}} \psi F(v) f(x) dx} = \frac{\psi F(v)(1-F(v))E[w|w>v]}{\eta + \psi F(v)(1-F(v))}.$$

We can use the fact that $U(v, v)$ is continuous and $U(\overline{v}, \overline{v}) = U(\underline{v}, \underline{v}) = 0$ to establish that a strictly positive mass of agents with *extreme pizzazz values marry* in finite time. The next comparative statics result follows directly from (5) and shows that not all agents benefit from modern, less frictional, matching markets.[8]

PROPOSITION 2: *The share of agents who marry in a symmetric equilibrium is weakly increasing in η and weakly decreasing in ψ. Moreover, it converges to 0 as η goes to zero, for any $\psi > 0$.*

Proposition 2 establishes that the share of eternal singles increases when the market becomes less frictional. Note that an agent v marries in symmetric equilibrium only if her/his opportunity value is at least v. Thus, if v marries, (s)he must believe that all agents of the opposite sex accept her/him with probability $\psi F(v)$ or higher. When frictions become less intense, waiting becomes less costly. If v thinks that top agents are achievable with probability $\psi F(v)$, (s)he is unwilling to accept agents of her/his caliber or lower. For similar reasons, v will never be accepted by agents of her/his caliber or higher, which makes it impossible for v to marry when the market becomes less and less frictional.

Proposition 2 also establishes that the market collapses when the search frictions vanish for any $\psi > 0$. Under rational expectations, the equilibrium outcomes in our model converge to the unique pairwise stable matching. This means that there is a discontinuity at $\psi = 0$: even slight departures from the rational expectations model can lead to radically different results when the meeting rate becomes sufficiently high.

Overoptimism and Oversearch. — In the previous section, we showed that some agents in our model may search indefinitely and never leave the market. We now explore the forces underlying this excessive search. We show that coarse reasoning

[8] In Section III, we show that a similar result holds when the distribution of singles is endogenous.

may lead to overoptimism, which, in turn, may lead to setting acceptance cutoffs that are too high. Furthermore, we find that there are two groups of agents who behave as if they were fully rational: one at the very top of the pizzazz distribution and one at the very bottom.

To understand the behavior of cursed agents, we first have to understand the mistakes they make. Cursed agents are correct about the average rate at which agents on the other side of the market accept them, i.e., $\int_{\underline{v}}^{\overline{v}} \sigma_x(v) f(x) dx = \int_{\underline{v}}^{\overline{v}} \gamma_v(x) f(x) dx$ for every v. However, because these cursed agents underestimate the extent to which agents with higher pizzazz value have higher standards, they overestimate the rate of mutual acceptance, i.e., $\int_{\hat{a}_v}^{\overline{v}} \sigma_x(v) f(x) dx < \int_{\hat{a}_v}^{\overline{v}} \gamma_v(x) f(x) dx$ whenever $\hat{a}_v > \underline{v}$, unless $a_v \in \{\underline{v}, \overline{v}\}$, in which case v is treated equally by everyone and makes no mistake. As a result, unless $a_v \in \{\underline{v}, \overline{v}\}$ or $\hat{a}_v = \underline{v}$, agent v underestimates the time it it will take her/him to get married.

Unfortunately, the above mistake is not the only one cursed agents make. Given an acceptance cutoff $\hat{a}_v$, a cursed agent v falsely expects to marry someone whose pizzazz value is in $[\hat{a}_v, \overline{v}]$. However, if v marries (which is not guaranteed, as established in Proposition 2), then (s)he marries an agent with pizzazz value in $[\hat{a}_v, a_v]$, where $a_v \leq \overline{v}$. Thus, unless $a_v \in \{\overline{v}, \underline{v}\}$, agent v overestimates the payoff (s)he will obtain in a future marriage.

We can conclude that, unless $a_v \in \{\underline{v}, \overline{v}\}$, agent v overestimates her/his prospects in the market. The next result formalizes the above intuitions and shows that this overoptimism leads to oversearch and delay in matching for moderately desirable agents.

PROPOSITION 3: *Let $[v_1, \overline{v}]$ be the top class in Proposition 1. In a partially cursed equilibrium, every agent $v \in [v_1, \overline{v}]$ behaves as if (s)he were rational. If $v_1 > \underline{v}$, then there exists a threshold $v_2 \in (\underline{v}, v_1)$ such that every agent $v \in [\underline{v}, v_2)$ behaves as if (s)he were rational, while every agent $v \in [v_2, v_1)$ searches longer than a rational agent would.*

Agents who are accepted by all other agents are unaffected by cursedness as all other agents treat them equally. They correctly estimate both the expected time it will take them to marry and their future spouse's expected pizzazz. Thus, they behave as if they were fully rational. As a result the top class under partial cursedness is identical to the top class under rational expectations.

Agents at the bottom of the pizzazz distribution who accept all other agents never reject any matches and, in particular, they never reject matches that a fully rational agent would accept. Therefore, despite their overoptimism, they behave as if they were fully rational.

All other agents overestimate the expected pizzazz of their future spouse and underestimate the time it will take them to get married. Therefore, they use an acceptance cutoff that is higher than the one a rational agent would use. In other words, due to their overoptimism about their prospects in the market, they reject matches that a fully rational agent would accept.

Characterization and Existence. — In this section, we construct a symmetric equilibrium in which there is block segregation. Our construction shows that, in general, there are an infinite number of such equilibria. However, by Condition 4, the set of agents who marry in a symmetric equilibrium is unique.

In constructing the symmetric equilibria, we use the fact that by Condition 4 $[\underline{v}, \overline{v}]$ is partitioned into maximal intervals in which either all agents marry or none do. We refer to these intervals as marriage intervals and singles intervals, respectively. The transition between intervals occurs at points v such that $U(v, v) = v$. We treat each interval separately and partition it into a potentially infinite number of *classes*, in which all agents share the same acceptance cutoff and opportunity value.

As in the rational expectations case, a top class exists and it is possible to construct a sequence of classes starting from this class. However, unlike in the rational expectations case, the sequence will not necessarily cover $[\underline{v}, \overline{v}]$. We show that when it does not, it converges to the highest pizzazz v such that $U(v, v) = v$. That is, these classes cover only the top marriage interval.

The main challenge in the proof is that in any other interval but the top one there is no upper class from which we can start the construction. Nevertheless, we show that it is possible to define an arbitrary initial class in the interior of each interval and construct two unique sequences of classes on each of the initial class's sides. The sequences cover the interval and converge to its endpoints. The arbitrariness in defining the initial class implies that there are an infinite number of equilibria whenever $[\underline{v}, \overline{v}]$ is partitioned into more than one interval.

Formally, by Condition 4, we can partition $[\underline{v}, \overline{v}]$ into maximal intervals in which agents either eventually marry or remain single forever. We say that L is a *marriage interval* if L is a maximal interval such that $U(l, l) < l$ for all $l \in L$. An interval L is said to be a *singles interval* if either L is a maximal interval such that $U(l, l) > l$ for all $l \in L$, or $U(l, l) = l$ for all $l \in L$. In the latter case, L is often a singleton. Denote the closure of L, $cl(L)$, by $[\underline{l}, \overline{l}]$.

A *class* is a nondegenerate interval in which agents have identical acceptance cutoffs and opportunity values. In classes contained in marriage intervals, or "*marriage classes*," agents' acceptance cutoffs are equal to the class's infimum and their opportunity values are equal to its supremum. In classes contained in singles intervals, or "*singles classes*," agents' acceptance cutoffs are equal to the class's supremum and their opportunity values are equal to its infimum. Formally, a *class* is a nonempty interval C such that for every $v, w \in C$, it holds that $a_v = a_w$, $\hat{a}_v = \hat{a}_w$ and $\{a_v, \hat{a}_v\} = \{\underline{c}, \overline{c}\}$, where $[\underline{c}, \overline{c}] = cl(C)$ and $\underline{c} \neq \overline{c}$.

The following lemma is key in establishing that, in equilibrium, if a marriage/singles interval contains one class, then it is covered by classes.

LEMMA 3: *In equilibrium, if a marriage/singles interval L contains a class C, then (i) unless $\overline{c} = \overline{v}$, L contains a unique class C' such that $\overline{c} = \underline{c'}$, and (ii) unless $\underline{c} = \underline{v}$, L contains a unique class C'' such that $\overline{c''} = \underline{c}$.*

By Lemma 3, if a marriage/singles interval L contains a finite number of classes, then $L = [\underline{v}, \overline{v}]$. Moreover, an infinite sequence of adjacent classes must converge to some $v^\star$ satisfying $U(v^\star, v^\star) = v^\star$. To see this, let C_n be an infinite sequence of adjacent classes and note that $\underline{c_n}$ and $\overline{c_n}$ both converge to the same pizzazz value, v. As a result, both $U(\underline{c_n}, \overline{c_n})$ and $U(\overline{c_n}, \underline{c_n})$ converge to $U(v, v)$. Moreover, by optimality, $U(\underline{c_n}, \overline{c_n}) = \underline{c_n}$ in marriage classes and $U(\overline{c_n}, \underline{c_n}) = \overline{c_n}$ in singles classes, and so $U(\underline{c_n}, \overline{c_n})$ and $U(\overline{c_n}, \underline{c_n})$ converge to v. Thus, $U(v, v) = v$. We can conclude that for any marriage/singles interval L, it holds that if $\underline{l} \neq \underline{v}$ then $\underline{l} = U(\underline{l}, \underline{l})$ and if $\overline{l} \neq \overline{v}$ then $\overline{l} = U(\overline{l}, \overline{l})$. The next corollary follows immediately.

COROLLARY 1: *If a marriage/singles interval contains one class, then it is covered by classes.*

Due to the search frictions, the agents' continuation values, and hence their optimal acceptance cutoffs, are bounded away from $\overline{v}$. Thus, in equilibrium, there are two sets of agents, one on each side of the market, who are accepted by every agent of the opposite sex. All of these agents have the same continuation value and, therefore, they use the same acceptance cutoff, which, in turn, defines the set of agents who are accepted by all the other agents on the other side of the market. These agents form the top class and are uniquely determined by the primitives of the model. Thus, Corollary 1 and the existence of an upper class (Proposition 3) imply the following corollary.

COROLLARY 2: *In equilibrium, the top marriage interval is covered by classes in a unique manner.*

If η is sufficiently large such that $U(v, v) < v$ for all v, implying that all agents marry in equilibrium, then, by Corollary 2, the equilibrium is unique. Otherwise, the equilibrium is uniquely defined only in the top marriage interval.

In the following proposition, we show equilibrium existence by construction.[9]

PROPOSITION 4: *There exists $\overline{\eta}$ such that if $\eta > \overline{\eta}$, then there exists a unique equilibrium and all agents marry and, if $\eta < \overline{\eta}$, then there exist an infinite number of equilibria, in each of which there is a set of agents who remain single forever, which is identical across equilibria.*

DISCUSSION: SYMMETRY IN THE MODEL. — Throughout the analysis, we focused on symmetric equilibria and studied the case where the men's and women's levels of strategic sophistication are identical. These assumptions allowed us to convey the main messages succinctly. However, our key insights are not sensitive to these assumptions. We now discuss the implications of relaxing these assumptions.

[9] In the equilibria we construct, all agents are partitioned into classes. However, when $\eta < \overline{\eta}$, there are additional equilibria in which the top marriage interval is covered by classes while in every other interval there are no classes at all. Thus, there are equilibria in which close "types" do not have completely disjointed sets of potential partners except in the top marriage interval.

Asymmetric Equilibria

The cornerstone of our analysis of symmetric equilibria was the necessary and sufficient condition for marriage, $U(v,v) < v$. In asymmetric equilibria, this condition is necessary for marriage, but it is no longer sufficient. Thus, the agents who marry in a symmetric equilibrium form a superset of the agents who marry in any asymmetric equilibrium.

When $U(v,v) > v$, agent v cannot marry in asymmetric equilibria. To see why, note that if a man (woman) v accepts a match with a woman (man) w, then $a_w \geq v$. Thus, $U(\hat{a}_w, a_w) \geq U(v, a_w) \geq U(v,v) > v$ such that w prefers continuing searching to marrying v. Hence, agent v is rejected by every agent whom v accepts and remains single forever.

In order to illustrate that $U(v,v) < v$ is not sufficient for marriage in asymmetric equilibria, let $\eta < \overline{\eta}$ and denote the lowest v for which $U(v,v) = v$ by $v^\star$. Note that $v^\star > \underline{v}$ since $U(\underline{v}, \underline{v}) = 0$. Set $\hat{a}_v = v^\star$ for all women with pizzazz $v \in [\underline{v}, v^\star]$ and $\hat{a}_v = \underline{v}$ for all men with pizzazz $v \in [\underline{v}, v^\star]$. For all other agents, set acceptance cutoffs as in one of the symmetric equilibria constructed in the proof of Proposition 4. Note that (i) $a_v = v^\star$ for all women with pizzazz $v \in [\underline{v}, v^\star]$, which implies that $\hat{a}_v = v^\star$ is optimal for these women, and (ii) $a_v = \underline{v}$ for all men with pizzazz $v \in [\underline{v}, v^\star]$, which implies that $\hat{a}_v = \underline{v}$ is optimal for these men. As in the symmetric equilibrium, agents whose pizzazz is lower than $v^\star$ accept all agents whose pizzazz is higher than $v^\star$, which implies that the higher-pizzazz agents' behavior is optimal in the present case as well. Hence, the strategy profile we constructed is an equilibrium in which low-pizzazz agents never marry, in contrast to the symmetric equilibrium, in which these agents always do.

Asymmetric Strategic Sophistication

The assumption that both sides of the market are symmetric in their level of strategic sophistication is reasonable in the context of a marriage market. However, in other contexts, it makes sense to think that agents on different sides of the market differ in this respect. For instance, in the context of job search, employers engage in the market more frequently than job-seekers, which may lead to a better understanding of the market.

As a rough approximation of this idea, we now modify the baseline model by assuming that agents on one side of the market (women) are fully rational while agents on the other side of the market (men) are partially cursed. A natural question is whether the existence of fully rational agents on one side of the market alleviates the problem of oversearch. The next result shows that not only is the answer to this question negative, but, in fact, the share of eternal singles can increase on that side of the market.

PROPOSITION 5: *There exists a unique equilibrium. Relative to the case where all agents are partially cursed, in this equilibrium, it holds that*

- *The acceptance cutoffs, opportunity values, and share of men who marry in finite time are weakly higher.*
- *The acceptance cutoffs, opportunity values, and share of women who marry in finite time are weakly lower.*

In equilibrium, there is block segregation. The men's acceptance cutoffs pin down the women's classes and the women's acceptance cutoffs pin down the men's classes. However, since the women's and the men's levels of sophistication are different, the classes are asymmetric. The men's classes are just like in the rational expectations model (as they are pinned down by the behavior of fully rational women), and so all men marry in equilibrium. On the other hand, the women's classes are virtually identical to the ones obtained in the top marriage interval under partial cursedness (as they are pinned down by the behavior of partially cursed men) and, therefore, all women below the top marriage interval never marry in equilibrium (unlike in the partially cursed symmetric equilibrium in which some of these women do marry).

Note that fully rational women do not overestimate their prospects in the market. As a result, compared to the case where both sides of the market are partially cursed, they choose lower acceptance cutoffs. The women's lower cutoffs increase the men's opportunity values, which, in turn, makes the men increase their acceptance cutoffs. The increase in the men's acceptance cutoffs lowers the women's opportunity values and makes the women lower their acceptance cutoffs even further.

B. *The Analogy-Based Expectation Equilibrium*

In the previous section, we introduced a behavioral friction into the two-sided search framework. We assumed that agents' beliefs regarding the behavior of each individual on the other side of the market are affected by the average behavior of the entire population on that side, where the partial cursedness parameter allowed us to vary the magnitude of the agents' mistakes. We established that even a small departure from the rational expectations assumption can lead to extremely different outcomes.

In this section, we introduce a different behavioral friction. We assume that agents' beliefs regarding each individual on the other side of the market depend on the behavior of only a subset of agents on that side, whose pizzazz is similar to that individual's pizzazz. The size of these subsets allows us to capture the magnitude of the agents' mistakes: the smaller the subsets, the smaller the departure from the conventional model. We show that regardless of the size of these subsets, when the market becomes less frictional, there are fewer matches and more agents remain single forever.

We use the analogy-based expectation equilibrium (ABEE) (Jehiel, 2005) to incorporate this idea into the model. In an ABEE, players bundle different contingencies into exogenously given categories and fail to distinguish between the other players' behavior in different contingencies that belong to the same category. We adapt this concept by assuming that each agent divides the agents on the other side of the market into categories. The agent then believes that all agents who belong to the same category behave in the same manner. Specifically, each agent v believes that every member of a category accepts her/him as a partner with a probability equal to the average probability with which v is accepted by all of the category's members.

To illustrate the agents' beliefs in an ABEE, consider a woman w who is accepted by men whose pizzazz is lower than the median and rejected by all other men. If w were fully rational, then she would realize that only low-pizzazz men are willing to marry her. In an ABEE, when there is only one category, woman w thinks that all men are equally likely to accept her as a partner. Since half the men find her acceptable, she thinks that each man will accept her with probability 0.5. This case is equivalent to full cursedness (i.e., $\psi = 1$) as in Section II.A. Now, suppose that the men are partitioned into three adjacent categories of equal mass. Since all men in the bottom category accept woman w, she *correctly* believes that all men in this category accept her as a partner. Similarly, she correctly expects each man in the top category to reject her. However, woman w's beliefs are inaccurate with regard to men in the intermediate category, whom she expects to accept her with probability 0.5 regardless of their pizzazz.

As this example illustrates, unlike in the previous section, agents do not necessarily think that all other agents are achievable. When all agents in a specific category reject an individual, the latter understands that these agents are out of her/his league. The individual's beliefs are coarse only with respect to categories in which a fraction of the population accepts her/him.

Formally, we assume that the agents on each side of the market are partitioned into k adjacent cells of the same mass[10] $P_1, ..., P_k$. We denote $\overline{p}_j := sup(P_j)$ and $\underline{p}_j := inf(P_j)$. Every agent v believes that each $w \in P_j$ accepts her/him as a partner with probability β_{vj}. We say that a profile of beliefs $\beta = (\beta_{vj})_{v \in \mathcal{M} \cup \mathcal{W}, j \in \{1,...,k\}}$ is *consistent* with a profile of strategies σ if

$$\beta_{vj} = \frac{\int_{P_j \cap A_v(\sigma)} f(x)dx}{\int_{P_j} f(x)dx}$$

for every $v \in \mathcal{M} \cup \mathcal{W}$ and every cell $j \in \{1, ..., k\}$.

DEFINITION 2: *A profile of strategies σ and a profile of beliefs β form an ABEE if β is consistent with σ and, for every $v \in \mathcal{M} \cup \mathcal{W}$, σ_v is a best response to $(\beta_{vj})_{j \in \{1,...,k\}}$.*

In order to use the toolbox developed in the previous sections, we define $\gamma_v(w) := \beta_{vj}$ for $w \in P_j$. Thus, the expected discounted payoff given beliefs γ_v and accep-

[10]Our results are not sensitive to the assumption that each cell contains the same mass of agents.

tance cutoff $\hat{a}_v$, $U(\hat{a}_v, \gamma_v)$, is as given in (3).

The next lemma establishes that, in an ABEE, agents with higher pizzazz have higher standards. Its proof is identical to that of Lemma 1 and, therefore, omitted.

LEMMA 4: *In an ABEE, $\hat{a}_v$ and a_v are weakly increasing in v.*

Lemma 4 implies that all agents whose pizzazz is lower than a_v accept agent v. Since, by definition, agents whose pizzazz is higher than a_v reject v, there exists at most one cell in which different cell members treat v differently. Specifically, if $a_v \notin P_j$, then either all of the cell's members reject v or all of them accept v. Either way, v's beliefs about the cell's members' behavior are correct. On the other hand, if $a_v \in P_j$, then agent v's beliefs regarding that cell are coarse.[11] Hence, each agent holds accurate beliefs regarding the behavior of agents in at least $k-1$ cells. The larger k is, the larger the share of the population about whom the agents' estimates are accurate. Thus, k is a measure of the agents' mistakes. The next corollary formalizes this discussion.

COROLLARY 3: *In a symmetric ABEE, agent v correctly assesses the probability with which agent $w \in int(P_j)$ accepts her/him as a partner if and only if $a_v \notin int(P_j)$.*

Since agents' acceptance cutoffs are monotonic (Lemma 4), we can again write $U(\hat{a}_v, a_v)$ instead of $U(\hat{a}_v, \gamma_v)$ as we did in Section II.A. As in that section, U is weakly increasing in the opportunity value. Moreover, for every v such that $U(v, v) < v$ and every $w > v$, $U(\cdot, v)$ is weakly decreasing in its first argument at $U(w, v)$. Once this is established, the proof of Lemma 2 holds for ABEE (with a different expression for $U(\hat{a}_v, a_v)$ in (A1)). Lemma 5 states this formally.

LEMMA 5: *Agent v marries in a symmetric ABEE if and only if $U(v, v) < v$.*

We now derive the formula for $U(v, v)$. An agent who uses an acceptance cutoff $v \in P_j$ and whose opportunity value is v rejects all agents who belong to lower cells and, by Corollary 3, expects agents in higher cells to reject her/him. Thus, our agent understands that mutual acceptance is only possible when meeting agents who belong to P_j. Given an acceptance cutoff of v, the share of agents in P_j that our agent accepts is $k(F(\overline{p}_j) - F(v))$. The probability of meeting a member of P_j is $1/k$. Given an opportunity value of v, our agent expects to be accepted by each member of P_j with probability $k(F(v) - F(\underline{p}_j))$. Thus, our agent expects mutual acceptance with probability $k(F(v) - F(\underline{p}_j))(F(\overline{p}_j) - F(v))$ and, conditional on marriage, an expected payoff of $E[w|v < w < \overline{p}_j]$. Hence, for every $v \in P_j$,

$$U(v, v) = \frac{k(F(v) - F(\underline{p}_j))(F(\overline{p}_j) - F(v))E[w|v < w < \overline{p}_j]}{\eta + k(F(v) - F(\underline{p}_j))(F(\overline{p}_j) - F(v))}. \tag{6}$$

[11] If $a_v = \underline{p}_j$ or $a_v = \overline{p}_j$, then agent v perfectly understands the behavior of all agents with the possible exception of agent a_v.

Since $U(v, v)$ is continuous in v and $U(v, v) = 0$ at the cells' boundaries, the condition for marriage is always satisfied around the boundaries of each cell, and, in particular, at $\overline{v}$ and $\underline{v}$. Hence, only agents with interior pizzazz values may remain single forever.

In the next result, we use (6) and Lemma 5 to obtain comparative statics. As in the previous section, when the market becomes less frictional, the share of agents who search indefinitely increases and, when search frictions vanish, the market collapses.

PROPOSITION 6: *The share of agents who marry in a symmetric ABEE is weakly increasing in η and converges to* 0 *when η goes to* 0*.*

In an ABEE, the market collapses as search frictions vanish even though agents do not necessarily expect to marry agents who are significantly more desirable than themselves. To see this, note that when search frictions are sufficiently small, a_v and v belong to the same cell.[12] Thus, agents misestimate the probability with which they are accepted only with regard to agents who belong to the same cell as themselves. Hence, when search frictions vanish, the agents' expectations are realistic in the sense that they do not expect to marry agents who are out of their league. These expectations are different from the agents' beliefs in a partially cursed equilibrium, which assign strictly positive probability to marrying agents with extremely high pizzazz value.

While under both solution concepts the share of eternal singles increases when the market becomes less frictional, the characteristics of these eternal singles can be very different. In an ABEE, agents close to the boundary of each cell always marry (including cells that contain agents with intermediate pizzazz value). However, in a partially cursed equilibrium, when η is sufficiently small, only agents with extremely high or extremely low pizzazz value marry. Hence, for small values of η there is one singles interval under partial cursedness while the number of singles intervals in an ABEE is equal to the number of cells.

The Relation Between Partially Cursed and Analogy-based Expectation Equilibria

Miettinen (2009) shows that any partially cursed equilibrium of a Bayesian game corresponds to an analogy-based expectation equilibrium of the same game with an extended state space.[13] While the setting is different, a similar result holds in our model as well. Suppose that each agent v believes that a share of ψ of the agents on the other side of the market are *romantic* and a share of $1 - \psi$ are not, where being romantic is unobservable, payoff-irrelevant, and independent of pizzazz. The analogy partition consists of a continuum of singleton analogy classes, one for each unromantic type, and one analogy class for all the romantic

[12] With the possible exception of agents at a cell's boundaries.

[13] Formally, the equivalence Miettinen establishes is between Jehiel and Koessler (2008) version of ABEE and the partially cursed equilibrium.

types. Thus, in an ABEE agent v believes that an agent w on the other side of the market accepts her/him with probability $\psi \int_{\underline{v}}^{\overline{v}} \sigma_x(v) f(x) dx + (1-\psi)\sigma_w(v)$, exactly as in the partially cursed equilibrium.

III. Exogenous Inflow

Throughout the analysis, a stationary distribution of singles was maintained by the assumption that every agent who leaves the market is immediately replaced by an agent with the same pizzazz. In this section, we relax this assumption and let the flow of agents into the market be exogenous. To maintain the stationarity of the singles distribution, we require that, in equilibrium, the flow out of the market be equal to the flow into the market.

We follow Burdett and Coles (1997) by assuming that agents on both sides of the market enter at a flow rate β. New entrants are randomly drawn from a differentiable CDF G with a density function g that satisfies $0 < \underline{g} \leq g(v) \leq \overline{g} < \infty$ for any $v \in [\underline{v}, \overline{v}]$. Furthermore, as in Burdett and Coles (1997), we assume that the lifetime of each agent follows an exponential distribution with parameter $\delta > 0$; i.e., δdt is the probability that any agent dies in a short time interval dt. Adding the possibility of death to our baseline model changes only the expression for η (see (8)) and has no effect on any of the previous results.

While in the market, agents randomly meet singles of the opposite sex according to a quadratic search technology: given a distribution of singles F, agents meet potential partners at a rate $\mu F(\overline{v})$, where $F(\overline{v})$ is the mass of singles on each side of the market. This is consistent with our baseline model in which $F(\overline{v}) = 1$ and the meeting rate is μ. Meetings are random: agents meet agents of the opposite sex with pizzazz value in $[v_1, v_2]$ at a flow rate proportional to their mass in the population $\mu[F(v_2) - F(v_1)]$.

We assume that agents are partially cursed and extend the definition of partially cursed equilibrium to include the requirement that the distribution of singles F is in a steady state. In other words, in addition to the requirement that agents' strategies be optimal given their cursed beliefs, the flow of agents into and out of the market must be balanced.[14]

DEFINITION 3: *A partially cursed steady-state equilibrium is a profile of stationary strategies σ and a distribution F that satisfy the following conditions for every $v \in [\underline{v}, \overline{v}]$:*

- *Balanced flow*

$$\beta g(v) = f(v)\left(\delta + \mu \int_{\underline{v}}^{\overline{v}} \sigma_v(x)\sigma_x(v) f(x) dx\right). \tag{7}$$

[14]Generalizing the analogy-based expectation equilibrium is more involved and requires more notation. Nonetheless, it is possible to obtain the results in this section under a generalized ABEE as well.

- *Partially cursed beliefs*

$$\gamma_v(w) = \psi \frac{1}{F(\overline{v})} \int_{\underline{v}}^{\overline{v}} \sigma_x(v) f(x) dx + (1-\psi)\sigma_w(v).$$

- *Optimal strategies* $\hat{a}_v = max\{\underline{v}, U_v^\star\}$, *where*

$$U_v^\star = \frac{\int_{\hat{a}_v}^{\overline{v}} \gamma_v(x) x f(x) dx}{\eta + \int_{\hat{a}_v}^{\overline{v}} \gamma_v(x) f(x) dx}, \tag{8}$$

and $\eta = (r+\delta)/\mu$.

PROPOSITION 7: *A partially cursed steady-state equilibrium exists.*

The proof is essentially an adaptation of Proposition 1 in Smith (2006) that allows for cursed beliefs. Smith's singles market is slightly different from ours as, in his model, the inflow of agents into the market is via dissolution of existing marriages. However, as noted by Lauermann, Nöldeke and Tröger (2020), the balanced-flow conditions in both models are essentially equivalent. This observation implies that the fundamental matching lemma (Shimer and Smith, 2000), which Smith's proof is based on, holds in our setting as well.

Our equilibrium characterization results continue to hold in this general setting as they rely on the stationarity of the distribution of singles but not on how this stationarity is obtained. However, we can no longer use the tools developed to obtain comparative statics with respect to μ as changes in μ affect the distribution of singles in the market. The next result shows that, nonetheless, the main insight of the paper holds.

PROPOSITION 8: *The share of agents who marry in a symmetric partially cursed steady-state equilibrium converges to* 0 *as* μ *goes to* ∞.

In Section II.A, we established that a partially cursed agent will not settle for an agent of her/his caliber if (s)he expects to meet high-pizzazz agents of the opposite sex frequently enough. For a fixed distribution of singles, this effect occurs when μ is sufficiently large. In the steady-state model of this section, the distribution of singles depends on μ and, potentially, the mass of agents at the top of the pizzazz distribution can be smaller for large values of μ such that the rate at which singles meet high-pizzazz agents of the opposite sex does not necessarily go up with μ. The key challenge in the proof is to show that although the mass of agents with high pizzazz goes down when μ becomes large, the rate at which agents meet members of the top class, $\mu(F(\overline{v}) - F(\hat{a}_{\overline{v}}))$, goes to infinity when μ goes to infinity, which guarantees the desired result.

IV. Concluding Remarks

We studied a model of the marriage market in which the participants' reasoning is coarse. In equilibrium, agents who underestimate the correlation between desirability and selectivity overvalue their prospects in the market as they put too much weight on the possibility of marrying highly attractive individuals. As a result, they set standards that are too high and search longer than is optimal. This leads to prolonged singlehood and may even result in an eternal search. Our results imply that when agents are not fully rational, technological advances that thicken markets and enable faster and more efficient search can exacerbate the agents' biases and make them worse off overall.

Throughout the analysis we assumed that agents who marry obtain the pizzazz of their spouse or, in the words of Burdett and Coles (1997), "Looking in the mirror to admire one's own pizzazz does not increase utility." While this is natural in some contexts, in others there is some complementarity between partners. The main results and intuitions of the paper hold in many of these settings (e.g., when the payoff function is multiplicatively separable, as analyzed in Eeckhout 1999). In fact, as long as agents with higher pizzazz have higher standards, which is the case in any form of assortative mating, our qualitative results hold.

Although we use the marriage terminology in this paper, we wish to stress that the model and the main insights have implications for the labor market as well. As in the marriage market, new search technologies have changed the way people search for a job. For example, social networks such as LinkedIn enable employers and job-seekers to match faster than ever before. In the context of job search, additional factors may come into play as employers and potential hires can negotiate wages. However, as long as utility is not fully transferable and the job-seekers' preferences over employers are correlated, there will be some degree of vertical heterogeneity and our insights will remain valid.

We conclude by discussing a few extensions and modifications of the baseline model.

Market Segmentation

In practice, improvements in the matching algorithm allow individuals to meet more relevant people and fewer irrelevant people in a given time. Throughout the paper, we studied the implications of faster search, which is equivalent to meeting both more relevant *and* irrelevant individuals in a given time span, and established that when agents' reasoning is coarse faster search leads to overoptimism and oversearch. We now examine the effect of meeting fewer irrelevant people by segmenting the market such that individuals meet only people of relatively similar pizzazz.

When agents' reasoning is coarse, partitioning the market into smaller segments consisting of agents with similar pizzazz can help them obtain better outcomes.[15]

[15] When agents are fully rational, partitioning the market into segments coarser than the class partition

Segmentation reduces the negative externalities imposed by *irrelevant* agents of the opposite sex, bringing the agents' expectations closer to rational expectations. To see how, consider a woman w. When the market is segmented, she no longer meets men with significantly lower pizzazz whom she rejects but who accept her. This reduces the probability with which she believes that *other*, more desirable, men accept her. Second, she no longer meets men with significantly higher pizzazz whom she falsely believes that she can marry. This reduces her perceived expected value from future matches. Both effects reduce woman w's perceived value of remaining single and continuing to search.

In the Appendix, we formalize the above argument and establish that if a market with eternal singles is segmented, then their share strictly decreases. Moreover, the proof shows that agents' perceived continuation values are lower in the segmented market. Our analysis has two implications. On the one hand, segmentation can help agents obtain better outcomes when the speed of search is high. On the other hand, from the agents' naive perspective, segmentation reduces welfare, which could make segmented platforms less attractive to them.

Models of Overoptimism

One insight of this paper is that coarse reasoning can lead individuals to overestimate their prospects in the marriage market.[16] This overoptimism leads agents to oversearch and may result in some agents being eternal singles. A natural question that arises is whether other forms of overoptimism lead to similar effects. In general, overoptimism leads to delay in matching: agents who overestimate the prospect of remaining single typically reject matches a rational agent would accept. However, not every form of overoptimism leads to indefinite search.

Whenever agents perfectly understand which agents of the opposite sex are achievable they marry in finite time. For example, suppose that agents overestimate the rate at which they meet other singles; that is, they overestimate μ, but are otherwise fully rational. In this case, agents' equilibrium behavior is as in the standard rational expectations model, only with $\mu' > \mu$. Thus, there is a unique equilibrium and, in this equilibrium, all agents marry in finite time (although it may take them more time to marry than it would if they were to hold correct expectations).

Agents who assign a strictly positive probability to marrying agents who reject them may remain eternal singles. This occurs when agents' reasoning is coarse, but also in other models. As an illustration, consider a model in which agents overestimate their own pizzazz—each agent v falsely believes that her/his pizzazz is $min\{v + \Delta, \overline{v}\}$—but otherwise are fully rational. If the meeting rate is suffi-

has no effect on the equilibrium outcomes.

[16]This finding is consistent with the experimental literature. For example, in a comprehensive review, Camerer (1997) states that "dozens of studies show that people generally overrate the chance of good events, underrate the chance of bad events and are generally overconfident about their relative skill or prospects."

ciently high, in equilibrium, there are agents who will search indefinitely, believing that their opportunity value is higher than it really is.[17]

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[17]It is possible to show that, in equilibrium, there is an alternating sequence of classes: a marriage class followed by a singles class followed by a marriage class and so on. The size of the support of each singles class is Δ.

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Appendix: Proofs

Proof of Lemma 1. Let $v < w$. Since agents use cutoff strategies, it follows that $A_v \subseteq A_w$. Hence, $a_v \leq a_w$. Moreover, $A_v \subseteq A_w$ also implies that $\gamma_v(x) \leq \gamma_w(x)$ for every $x \in [\underline{v}, \overline{v}]$. Hence, if agent w were to use agent v's optimal acceptance cutoff, then w would obtain a perceived expected discounted payoff of at least $U_v^\star$. Hence, $U_w^\star \geq U_v^\star$ and, thus, $\hat{a}_w \geq \hat{a}_v$.

Proof of Lemma 2. For any acceptance cutoff $\hat{a}_v$ and opportunity value a_v,

$$U(\hat{a}_v, a_v) = \frac{\int_{\hat{a}_v}^{\overline{v}} (\psi F(a_v) + (1-\psi)\sigma_x(v)) x f(x) dx}{\eta + \int_{\hat{a}_v}^{\overline{v}} (\psi F(a_v) + (1-\psi)\sigma_x(v)) f(x) dx}. \tag{A1}$$

In a symmetric equilibrium, if agent v rejects agents whose pizzazz is v, then v is rejected by agents with pizzazz v on the other side of the market as well. That is, $\hat{a}_v > v$ implies $a_v \leq v$. By the same logic, $\hat{a}_v \leq v$ implies $a_v \geq v$. Note that agent v marries in finite time if and only if $\hat{a}_v < a_v$. Thus, agent v marries in finite time if and only if $\hat{a}_v \leq v \leq a_v$, with at least one strict inequality.

To show necessity, let $U(v, v) \geq v$ and consider an equilibrium. First, if $a_v < v$, then the necessary condition for marriage above is violated. Second, if $a_v \geq v$, then $\hat{a}_v \geq U_v^\star = U(\hat{a}_v, a_v) \geq U(v, a_v) \geq U(v, v) \geq v$, where the second inequality follows from $\hat{a}_v$ being optimal and the third one follows from (A1) being increasing in its second argument. This contradicts the necessary condition for marriage.

To show sufficiency, let $U(v, v) < v$. We start by assuming that $v > \underline{v}$ and take care of $v = \underline{v}$ later. Assume to the contrary that v does not marry, that is, $a_v \leq \hat{a}_v$. In a symmetric equilibrium, $a_v \leq \hat{a}_v$ implies that $a_v \leq v \leq \hat{a}_v$. Optimality of $\hat{a}_v$ implies that $\hat{a}_v = max\{\underline{v}, U(\hat{a}_v, a_v)\}$. Since $\hat{a}_v \geq v > \underline{v}$, it holds that $U(\hat{a}_v, a_v) = \hat{a}_v \geq v$. Since (A1) is increasing in its second argument, $U(\hat{a}_v, v) \geq v$. Moreover, since $U(w, v)$ is decreasing in its first argument for any $w > v$ if $U(v, v) < v$, it holds that $U(v, v) \geq U(\hat{a}_v, v) \geq v$, which violates the assumption that $U(v, v) < v$.

We now show that agent $\underline{v}$ marries in equilibrium (note that $U(\underline{v}, \underline{v}) < \underline{v}$). Assume to the contrary that $\underline{v}$ remains single and, therefore, $a_{\underline{v}} \leq \underline{v}$. Thus, $\hat{a}_v > \underline{v}$ and $U_v^\star > \underline{v}$ for every $v > \underline{v}$. Denote by z the opportunity value that induces both an optimal acceptance cutoff and continuation value $\underline{v}$. Since $\underline{v} > 0$, it follows that $z > \underline{v}$. Note that for any $v \in (\underline{v}, \hat{a}_z)$, it holds that $a_v \leq z$. Thus, for any such v, $U_v^\star \leq \underline{v}$, in contradiction to $\hat{a}_v > \underline{v}$ being part of an equilibrium. We can conclude that $a_{\underline{v}} \neq \underline{v}$. Thus, $a_{\underline{v}} > \underline{v}$. By symmetry, $\hat{a}_{\underline{v}} = \underline{v}$ and the sufficient condition for marriage $a_{\underline{v}} > \hat{a}_{\underline{v}}$ holds.

Proof of Proposition 2. From (5), we can see that $U(v, v)$ is strictly increasing in ψ and strictly decreasing in $\eta = r/\mu$. Moreover, at the $\eta = 0$ limit, $U(v, v)$ converges to $E[w|w \geq v] > v$ for all $v \in (\underline{v}, \overline{v})$. Thus, the share of agents who satisfy Condition 4 is decreasing in ψ, increasing in η, and converges to 0 when η goes to 0.

Proof of Proposition 3. This proof consists of three steps. First, we show that every agent $v \in [v_1, \overline{v}]$ behaves as if (s)he were fully rational. Second, we establish the existence of a threshold $v_2 > \underline{v}$ such that $\hat{a}_v = \underline{v}$ for every $v < v_2$. Lastly, we show that agents whose pizzazz is lower than v_1 overestimate the prospect of remaining single and, if their pizzazz is also greater than v_2, they oversearch. If their pizzazz value is below v_2, then they behave as if they were fully rational.

Due to the search frictions, $U^{\star}_{\overline{v}} < \overline{v}$. Hence, $\hat{a}_{\overline{v}} < \overline{v}$. Denote $v_1 := \hat{a}_{\overline{v}}$ and consider $v \geq v_1$. By Lemma 1, v is accepted by all agents of the opposite sex, and so $\gamma_v(w) = \sigma_w(v) = 1$ for any w. Thus, v forms correct expectations and, as a result, behaves as if (s)he were fully rational. Since a cutoff of $\hat{a}_v = v_1$ is optimal given $a_v = \overline{v}$ (both under rational expectations and under partial cursedness), it follows that $[v_1, \overline{v}]$ is the top class in Proposition 1.

In order to establish the threshold v_2, recall that $U(\underline{v}, \underline{v}) = 0 < \underline{v}$. Thus, agents at the bottom of the distribution marry in equilibrium. Hence, $\hat{a}_v = \underline{v}$ for some $v > \underline{v}$. By Lemma 1, there exists a maximal $v_2 > \underline{v}$ such that $\hat{a}_v = \underline{v}$ for every $v < v_2$.

Next, we show that if $v_2 < v_1$, then agents whose pizzazz is $v \in [v_2, v_1)$ oversearch and that agents whose pizzazz is lower than v_2 behave as if they were fully rational.

Let v be such that $a_v > \hat{a}_v$. Consider agent v's perceived probability of marriage. Agent v believes that, conditional on accepting a match, (s)he will be accepted with probability

$$\psi F(a_v) + (1 - \psi) \frac{F(a_v) - F(\hat{a}_v)}{1 - F(\hat{a}_v)}. \tag{A2}$$

However, conditional on accepting a match, v is accepted with probability

$$\frac{F(a_v) - F(\hat{a}_v)}{1 - F(\hat{a}_v)},$$

which is smaller than (A2) unless $\hat{a}_v = \underline{v}$ or $a_v = \overline{v}$ (in either case, the two expressions are equal and agent v correctly estimates this probability). Thus, if $v \in [v_2, v_1)$, then (s)he underestimates the time it will take her/him to marry.

Now consider agent v's perceived expected payoff from marriage. Agent v will marry an agent whose expected pizzazz is $E[w|\hat{a}_v \leq w < a_v]$. However, v believes that (s)he will marry an individual whose expected pizzazz is

$$\frac{\psi F(a_v)(1 - F(\hat{a}_v))E[w|\hat{a}_v \leq w] + (1 - \psi)(F(a_v) - F(\hat{a}_v))E[w|\hat{a}_v \leq w < a_v]}{\psi F(a_v)(1 - F(\hat{a}_v)) + (1 - \psi)(F(a_v) - F(\hat{a}_v))},$$

which is higher than $E[w|\hat{a}_v \leq w < a_v]$, unless $a_v = \overline{v}$, in which case the two expressions are equal. Thus, if $v < v_1$, then (s)he overestimates the expected pizzazz of her/his eventual partner.

We have seen that, unless $a_v = \overline{v}$, in which case v is correct, agent v's perceived discounted expected payoff, $U_v^\star$, is higher than the actual one. Since $\hat{a}_v \in \{U_v^\star, \underline{v}\}$, whenever the agent chooses an acceptance cutoff $\hat{a}_v > \underline{v}$, it is too high as well. It follows that every $v \in [v_2, v_1)$ searches longer than a rational agent would. Finally, if setting a cutoff $\hat{a}_v = \underline{v}$ is optimal given $U_v^\star$, then it is also optimal given the correct expected payoff, which is weakly lower. Hence, every agent whose pizzazz is $v \leq v_2$ behaves as if (s)he were fully rational.

To complete the proof, let v be such that $\hat{a}_v \geq a_v$. Recall that $a_{\underline{v}} > \underline{v}$ and, by monotonicity, $a_v > \underline{v}$ for all $v > \underline{v}$. Thus, agent v can marry by setting a low enough cutoff. However, agent v marries with probability 0 and gains an actual expected payoff of 0. Thus, the agent's acceptance cutoff is higher than optimal and the agent searches longer than a rational agent would given a_v.

Proof of Lemma 3. Assume that C is a marriage class and $\overline{c} \neq \overline{v}$. By the definition of a class, $\hat{a}_v \geq \overline{c}$ for any agent $v > \overline{c}$. Since C is in a marriage interval, $\lim_{v \to \overline{c}^+} \hat{a}_v = \overline{c}$. There exists a unique pizzazz value $a_{\overline{c}} > \overline{c}$ such that an acceptance cutoff of $\overline{c}$ is optimal given an opportunity value $a_{\overline{c}}$. Thus, $a_{\overline{c}} = \lim_{v \to \overline{c}^+} a_v$, and, as a result, $\hat{a}_{a_{\overline{c}}} = \overline{c}$. By monotonicity, $\hat{a}_v = \overline{c}$ for any $v \in (\overline{c}, a_{\overline{c}})$. This implies that $a_v = a_{\overline{c}}$ for all such v. Therefore, $C' = [\overline{c}, a_{\overline{c}})$ is a class.

Assume that C is a singles class and note that $\overline{c} \neq \overline{v}$. For any $v > \overline{c}$, it holds that $a_v \geq \overline{c}$. Since C is in a singles interval, $\lim_{v \to \overline{c}^+} a_v = \overline{c}$. There exists a unique acceptance cutoff $\hat{a}_{\overline{c}} > \overline{c}$ that is optimal given an opportunity value $\overline{c}$. By monotonicity, $a_v = \overline{c}$ for any $v \in (\overline{c}, \hat{a}_{\overline{c}})$. This implies that $\hat{a}_v = \hat{a}_{\overline{c}}$ for all such v. Therefore, $C' = [\overline{c}, \hat{a}_{\overline{c}})$ is a class.

The proofs of the existence of C'' follow the same logic and are omitted for brevity.

Proof of Proposition 4. We consider each marriage/singles interval separately and define, for every agent v, an acceptance cutoff $\hat{a}_v$. For each agent v such that $U(v, v) = v$, set $\hat{a}_v = v$. In the remainder of the proof, singles intervals are assumed to contain only agents with $U(v, v) > v$.

First, we show that, for any opportunity value $a \in L$, there exists an acceptance cutoff $\hat{a} \in L$ such that $\hat{a}$ is optimal given a. Let L be a marriage/singles interval such that $\underline{v} \notin L$, and let $a \in L$. If L is a marriage interval, then $U(a, a) - a < 0$ and, since $U(\underline{l}, \underline{l}) = \underline{l}$, it follows that $U(\underline{l}, a) - \underline{l} > 0$. If L is a singles interval, then $U(a, a) - a > 0$ and, since $U(\overline{l}, \overline{l}) = \overline{l}$, it follows that $U(\overline{l}, a) - \overline{l} < 0$. In either case, since U is continuous, there exists $\hat{a} \in L$ such that $U(\hat{a}, a) = \hat{a}$. To complete this step, consider L such that $\underline{v} \in L$ and note that it is a marriage interval. Applying the argument above we get that either there exists $\hat{a}$ such that $U(\hat{a}, a) = \hat{a}$, or

$U(\hat{a}, a) < \hat{a}$ for all $\hat{a} \in L$. In the latter case, $\underline{v}$ is optimal given a.

Second, we show that, for any acceptance cutoff $\hat{a} \in L$, such that L is not the top marriage interval, there exists an opportunity value $a \in L$ that "rationalizes" it. Let L be a marriage/singles interval, and let $\hat{a} \in L$. If L is a marriage interval, then, the value w that maximizes $U(w, \hat{a})$ satisfies $\underline{l} < w < \hat{a}$, and the value w that maximizes $U(w, \bar{l})$ is $w = \bar{l} > \hat{a}$. By continuity, there exists an $a \in (\hat{a}, \bar{l})$ such that the value w that maximizes $U(w, a)$ is $w = \hat{a}$. If L is a singles interval, then the value w that maximizes $U(w, \hat{a})$ satisfies $w > \hat{a}$, and the value w that maximizes $U(w, \underline{l})$ is $w = \underline{l} < \hat{a}$. By continuity, there exists an $a \in (\underline{l}, \hat{a})$ such that the value w that maximizes $U(w, a)$ is $w = \hat{a}$. That is, in both cases, a rationalizes the acceptance cutoff $\hat{a}$.

Third, we use the insights from the first two steps to cover an arbitrary marriage/singles interval with classes. Let L be a marriage/singles interval that does not contain $\underline{v}$ or $\overline{v}$, and let $c^0 \in L$. For $k = 1, 2, ...$, let c^k be the pizzazz for which $U(c^{k-1}, c^k) = c^{k-1}$. For $n = 1, 2, ...$, let c^n be the pizzazz for which $U(c^{-n}, c^{1-n}) = c^{-n}$. Note that both series $\{c^k\}_{k \in \mathbb{N}}$ and $\{c^n\}_{n \in \mathbb{N}}$ are bounded and monotonic, and hence converge to k^* and l^*, respectively. At each of these limits, $v^* \in \{k^*, l^*\}$, it must hold that $v^* = a_{v^*} = \hat{a}_{v^*}$. Thus, $v^* \in \{\underline{l}, \bar{l}\}$. For $k \in \mathbb{Z}$, define $C^k = [c^k, c^{k+1})$. Note that the sets $\{C^k\}_{k \in \mathbb{Z}}$ are disjoint and cover L. Set $\hat{a}_v = c^k$ for any $v \in [c^k, c^{k+1})$, $\hat{a}_{\underline{l}} = \underline{l}$, and $\hat{a}_{\bar{l}} = \bar{l}$.

Let L be a marriage interval containing $\overline{v}$. Set $c^0 = \overline{v}$. For any $n = 1, 2, ...$, let c^n be the pizzazz for which $U(c^{-n}, c^{1-n}) = c^{-n}$. In the case of $c^{-n} \leq \underline{v}$, set $c^{-n} = \underline{v}$ and stop the process. Define $C^n = [c^{-n}, c^{-n+1})$ for $n = 1, 2, ...$. As in the previous case, the sets $\{C^{-n}\}_{n \in \mathbb{N}}$ are disjoint and cover L. Set $\hat{a}_v = c^{-n}$ for any $v \in [c^{-n}, c^{1-n})$ and $\hat{a}_{\overline{v}} = c^{-1}$.

Let L be a marriage interval containing $\underline{v}$ but not $\overline{v}$. Set $c^0 = \underline{v}$. For any $k = 1, 2, ...$, let c^k be the pizzazz for which $U(c^{k-1}, c^k) = c^{k-1}$. Define $C^{k-1} = [c^{k-1}, c^k)$ for any $k = 1, 2....$ Set $\hat{a}_v = c^{k-1}$ for any $v \in [c^{k-1}, c^k)$.

By observing the implied opportunity values, it is straightforward to verify that the acceptance cutoffs that we defined form an equilibrium. By construction, $a_v = v$ for any v for which $v = U(v, v)$. If $v \neq U(v, v)$, then in any marriage/singles interval L, if $\hat{a}_v = c^k$ for some k, then $a_v = c^{k+1}$. By construction, all acceptance cutoffs $\hat{a}_v$ are optimal given their respective opportunity values a_v. Thus, an equilibrium exists. The existence of a cutoff $\overline{\eta}$ is implied by Proposition 2.

Proof of Proposition 5. Consider a symmetric equilibrium in which both sides of the market are partially cursed (with the same ψ) and let $\{C_p^k\}_k$ be the classes in its top marriage interval. Similarly, consider a symmetric equilibrium in which both sides are fully rational and let $\{C_r^k\}_k$ be the classes in this equilibrium. Note that $C_p^1 = C_r^1$ since the top class is not affected by cursedness. Assume that for $k \in \{1, ..., n-1\}$, all men in C_r^k accept women in C_p^k or higher, and all women in C_p^k accept men in C_r^k or higher. Note that if a woman is accepted by all men in

C_r^n or below and rejected by men in higher classes, then her best response is to accept men in class C_r^n and above. Similarly, if a man is accepted by all women in C_p^n or below and rejected by women in higher classes, then his best response is to accept women in class C_p^n and above.

Recall that there is a finite number of classes in $\{C_r^k\}_k$ (Proposition 1) and denote it by N. If $\underline{v} \notin C_p^N$, then all women whose pizzazz values are below C_p^N are not accepted by any men, and, therefore, they set their acceptance cutoff to $\underline{v}$. By construction, the strategies we have defined constitute the unique equilibrium in this model.

Note that given any opportunity value a_v, a rational agent chooses a weakly lower acceptance cutoff than a cursed agent.[18] Since agents' acceptance cutoffs are increasing in their opportunity value, women's classes $\{C_p^k\}_k$ are "higher" than men's classes $\{C_r^k\}_k$. That is, $inf(C_p^k) \geq inf(C_r^k)$ for all k.

If a women is in the k-th class in the asymmetric model, then she is also in the k-th class in the partially cursed symmetric equilibrium. However, in the asymmetric model, women in the partially-cursed-k-th class marry men in the rational-expectations-k-th class, which is "lower" than the partially-cursed-k-th class. Thus, a woman in the k-th class in the asymmetric model has weakly lower acceptance cutoff and opportunity value than in the partially cursed symmetric model.

Since men's classes have shifted downwards in the asymmetric model compared to the partially cursed symmetric equilibrium (i.e., $inf(C_p^k) \geq inf(C_r^k)$ for all k), each man now belongs to a weakly higher class. While the classes themselves have changed, the acceptance cutoff and opportunity values of men who belong to the k-th class have not, as they are pinned down by the women's classes. Thus, men's acceptance cutoffs and opportunity values are weakly higher in the asymmetric case.

Note that all men marry in equilibrium of the asymmetric model. As for women, if in a symmetric partially cursed equilibrium a woman w does not marry in finite time, then she is not part of the top marriage interval and $w < inf(C_p^N)$. Hence, as our construction shows, she cannot marry in the asymmetric case either. Thus, the share of women (respectively, men) who never marry is weakly higher (respectively, lower) in the asymmetric case.

Proof of Proposition 6. Consider (6) and observe that $U(v, v)$ is decreasing in η. Moreover, when η goes to 0, $U(v, v)$ goes to $E[w|v \leq w \leq \overline{p}_j] > v$ for $v \in int(P_j)$. Thus, the share of agents for whom the necessary and sufficient condition for marriage is satisfied becomes smaller when η decreases and it converges to 0 when η goes to 0.

Proof of Proposition 7. The arguments in this proof follow Proposition 1 in

[18]The inequality is strict except for the cases of $a_v = \overline{v}$ and $\hat{a}_v = \underline{v}$.

Smith (2006) with some necessary adaptation to our setting. Agents' optimal acceptance cutoffs, given their perceived continuation values, together with the inflow distribution of agents, determine the distribution of singles in the market, and induce a profile of perceived continuation values. This is essentially a mapping from one profile of continuation values to another. In the proof, an appropriate set of profiles of continuation values is defined, and it is shown that this mapping satisfies the conditions for Schauder's fixed-point theorem. This establishes that a steady-state equilibrium exists.

Let $\mathcal{U}$ be the set of measurable functions U on $[\underline{v},\overline{v}]$ that satisfy $0 \leq U_v \leq \overline{v}$ for every $v \in [\underline{v},\overline{v}]$ and have a uniformly bounded variation $B < \infty$. These value functions are integrable, and so, by Alaoglu's theorem, $\mathcal{U}$ is weak-* compact.

Denote by $\alpha^U(v_1, v_2) : [\underline{v},\overline{v}]^2 \to \{0,1\}$ the indicator function that specifies who marries whom given a profile of perceived continuation values U, that is, $\alpha^U(v_1,v_2) = \sigma^U_{v_1}(v_2) * \sigma^U_{v_2}(v_1)$, where σ^U is the agents' profile of strategies given U and $\sigma^U_v(w) = 1$ if $w \geq U_v$. Denote by f^α the density function of singles that satisfies the balanced-flow condition given α. Note that the balanced-flow condition implies that

$$0 < \frac{\beta g}{\frac{\beta}{\delta}\mu + \delta} \leq f \leq \frac{\beta g}{\delta}.$$

Endow α^U and f^α with $\mathcal{L}^1$ norms: $||\alpha||_{\mathcal{L}^1} = \int_{\underline{v}}^{\overline{v}}\int_{\underline{v}}^{\overline{v}} |\alpha(x,y)|dxdy$ and $||f||_{\mathcal{L}^1} = \int_{\underline{v}}^{\overline{v}} |f(x)|dx$.

LEMMA 6: *Part (i). Any Borel measurable map from value functions in $\mathcal{U}$ to match indicator functions is continuous. Part (ii). The map from match indicator functions to steady-state density solving* (7) *is both well defined and continuous.*

The lemma is essentially Lemma 8 in Smith (2006). The proof of Lemma 6 is the same as in Smith (2006) as it does not rely on the perceived continuation values U being correct. It is therefore omitted.

By Lemma 6, there exists a well-defined continuous mapping from value functions to densities solving the balanced-flow condition. We can therefore write the steady-state density, distribution, and cursed-belief functions given a profile U as f^U, F^U, and γ^U, respectively. In a steady-state equilibrium, (8) can be rewritten as

$$U_v = \frac{\int_{\underline{v}}^{\overline{v}} \gamma^U_v(x) max\{U_v, x\} f^U(x)dx}{\eta + \int_{\underline{v}}^{\overline{v}} \gamma^U_v(x) f^U(x)dx}, \tag{A3}$$

where

$$\gamma^U_v(x) = \frac{\psi}{F^U(\overline{v})}\int_{\underline{v}}^{\overline{v}} \sigma^U_x(v) f^U(x)dx + (1-\psi)\sigma^U_x(v).$$

Define the operator T on the set $\mathcal{U}$ as follows:

$$T(U_v) := \frac{\int_{\underline{v}}^{\overline{v}} \gamma_v^U(x) max\{U_v, x\} f^U(x)dx}{\eta + \int_{\underline{v}}^{\overline{v}} \gamma_v^U(x) f^U(x)dx}. \tag{A4}$$

Except for adjusting for the different function in (A4), the proofs of the next two lemmas are identical to the ones in Smith (2006).

LEMMA 7: *$T(U) \in \mathcal{U}$ for every $U \in \mathcal{U}$.*

Proof. By the definition of T, since $U \in [0, \overline{v}]$ for every $U \in \mathcal{U}$, it follows that $T(U) \in [0, \overline{v}]$. Moreover, T preserves measurability. It remains to show that the total variation of $T(U)$ is bounded by B for any $U \in \mathcal{U}$. Note that for $v_2 > v_1$,

$$T(U_{v_2}) - T(U_{v_1}) = \frac{\int_{\underline{v}}^{\overline{v}} \gamma_{v_2}^U(x) max\{U_{v_2}, x\} f^U(x)dx}{\eta + \int_{\underline{v}}^{\overline{v}} \gamma_{v_2}^U(x) f^U(x)dx} - \frac{\int_{\underline{v}}^{\overline{v}} \gamma_{v_1}^U(x) max\{U_{v_1}, x\} f^U(x)dx}{\eta + \int_{\underline{v}}^{\overline{v}} \gamma_{v_1}^U(x) f^U(x)dx}.$$

Following Smith (2006), we rewrite $T(U_{v_2}) - T(U_{v_1})$ as $Q_1(v_1, v_2) + Q_2(v_1, v_2)$, where

$$Q_1(v_1, v_2) = \frac{\int_{\underline{v}}^{\overline{v}} \gamma_{v_1}^U (max\{U_{v_2}, x\} - max\{U_{v_1}, x\}) f^U(x)dx}{\eta + \int_{\underline{v}}^{\overline{v}} \gamma_{v_1}^U(x) f^U(x)dx}$$

and

$$Q_2(v_1, v_2) = \frac{\int_{\underline{v}}^{\overline{v}} \gamma_{v_2}^U(x) max\{U_{v_2}, x\} f^U(x)dx}{\eta + \int_{\underline{v}}^{\overline{v}} \gamma_{v_2}^U(x) f^U(x)dx} - \frac{\int_{\underline{v}}^{\overline{v}} \gamma_{v_1}^U(x) max\{U_{v_2}, x\} f^U(x)dx}{\eta + \int_{\underline{v}}^{\overline{v}} \gamma_{v_1}^U(x) f^U(x)dx}.$$

Rearranging $Q_2(v_1, v_2)$ yields

$$\frac{\int_{\underline{v}}^{\overline{v}} (\gamma_{v_2}^U(x) - \gamma_{v_1}^U(x)) max\{U_{v_2}, x\} f^U(x)dx * (\eta + \int_{\underline{v}}^{\overline{v}} \gamma_{v_1}^U(x) f^U(x)dx)}{(\eta + \int_{\underline{v}}^{\overline{v}} \gamma_{v_2}^U(x) f^U(x)dx)(\eta + \int_{\underline{v}}^{\overline{v}} \gamma_{v_1}^U(x) f^U(x)dx)}$$

$$\frac{- \int_{\underline{v}}^{\overline{v}} \gamma_{v_1}^U max\{U_{v_2}, x\} f^U(x)dx * (\int_{\underline{v}}^{\overline{v}} (\gamma_{v_2}^U(x) - \gamma_{v_1}^U(x)) f^U(x)dx)}{(\eta + \int_{\underline{v}}^{\overline{v}} \gamma_{v_2}^U(x) f^U(x)dx)(\eta + \int_{\underline{v}}^{\overline{v}} \gamma_{v_1}^U(x) f^U(x)dx)}.$$

Note that $\int_{\underline{v}}^{\overline{v}} \gamma_v^U(x) f^u(x)dx = \int_{\underline{v}}^{\overline{v}} \sigma_x^U(v) f^U(x)dx$, $f^U \leq (\beta g)/\delta$ and $U_v \leq \overline{v}$. By the triangle inequality,

$$|Q_2(v_1, v_2)| \leq K\frac{\beta}{\delta}(G(a_{v_2}) - G(a_{v_1})), \tag{A5}$$

where

$$K = \frac{\overline{v}(\eta + 2\frac{\beta}{\delta})}{\eta^2}.$$

Let $\mathcal{P}$ be the set of partitions of $[\underline{v}, \overline{v}]$. By (A5), for any partition $\{v_j\} \in \mathcal{P}$,

$$\sum_{j=1}^{n} |Q_2(v_{j-1}, v_j)| \leq K \sum_{j=1}^{n} \frac{\beta}{\delta}(G(a_{v_j}) - G(a_{v_{j-1}})) = K\frac{\beta}{\delta}. \tag{A6}$$

To establish an upper bound for $|Q_1(v_1, v_2)|$, note that $|max\{a_2, b_2\}\text{-}max\{a_1, b_1\}| \leq |a_2 - a_1| + |b_2 - b_1|$, which implies

$$|Q_1(v_1, v_2)| \leq \frac{\int_{\underline{v}}^{\overline{v}} \gamma_{v_1}(x)|U_{v_2} - U_{v_1}|f(x)dx}{\eta + \int_{\underline{v}}^{\overline{v}} \gamma_{v_1}(x)f(x)dx} \leq \frac{\frac{\beta}{\delta}}{\eta + \frac{\beta}{\delta}}|U_{v_2} - U_{v_1}|. \tag{A7}$$

Denote the total variation of a function ϕ by $\langle\phi\rangle$. For any partition $\{v_j\} \in \mathcal{P}$,

$$\sum_{j=1}^{n} |Q_1(v_{j-1}, v_j)| \leq \frac{\frac{\beta}{\delta}}{\eta + \frac{\beta}{\delta}}\langle U\rangle. \tag{A8}$$

Hence,

$$\langle T(U)\rangle \leq K\frac{\beta}{\delta} + \frac{\frac{\beta}{\delta}}{\eta + \frac{\beta}{\delta}}\langle U\rangle. \tag{A9}$$

For a large enough B, $\langle U\rangle < B$ implies that $\langle T(U)\rangle \leq B$. We can conclude that $T(U) \in \mathcal{U}$ if $U \in \mathcal{U}$. ■

LEMMA 8: *The operator $T(U)$ is continuous.*

Proof. Continuity is proved by showing that $|\int_I T(U_v^1) - T(U_v^2)dv|$ is small whenever $|\int_I U_v^1 - U_v^2 dv|$ is small for any $I \subseteq [\underline{v}, \overline{v}]$ and any U^1 and U^2. By (A4), the expressions for $T(U_v^1)$ and $T(U_v^2)$ are different in three aspects: the density function, cursed-belief function, and value function. By adding and subtracting a series of expressions that differ in only one of these aspects each time, we can write $T(U_v^1) - T(U_v^2)$ as

$$\frac{\int_{\underline{v}}^{\overline{v}}(\gamma_v^{U^1}-\gamma_v^{U^2})(x)max\{U_v^1,x\}f^{U^1}(x)dx}{\eta+\int_{\underline{v}}^{\overline{v}}\gamma_v^{U^1}(x)f^{U^1}(x)dx}+$$

$$\frac{\int_{\underline{v}}^{\overline{v}}\gamma_v^{U^2}(x)(max\{U_v^1,x\}-max\{U_v^2,x\})f^{U^1}(x)dx}{\eta+\int_{\underline{v}}^{\overline{v}}\gamma_v^{U^1}(x)f^{U^1}(x)dx}+$$

$$\frac{\int_{\underline{v}}^{\overline{v}}\gamma_v^{U^2}(x)max\{U_v^2,x\}(f^{U^1}-f^{U^2})(x)dx}{\eta+\int_{\underline{v}}^{\overline{v}}\gamma_v^{U^1}(x)f^{U^1}(x)dx}+$$

$$\left(\frac{1}{\eta+\int_{\underline{v}}^{\overline{v}}\gamma_v^{U^1}(x)f^{U^1}(x)dx}-\frac{1}{\eta+\int_{\underline{v}}^{\overline{v}}\gamma_v^{U^2}(x)f^{U^2}(x)dx}\right)\int_{\underline{v}}^{\overline{v}}\gamma_v^{U^2}(x)max\{U_v^2,x\}f^{U^2}(x)dx.$$

Note that

$$\frac{1}{\eta+\int_{\underline{v}}^{\overline{v}}\gamma_v^{U^1}(x)f^{U^1}(x)dx}-\frac{1}{\eta+\int_{\underline{v}}^{\overline{v}}\gamma_v^{U^2}(x)f^{U^2}(x)dx}=$$

$$\frac{\int_{\underline{v}}^{\overline{v}}(\gamma_v^{U^2}-\gamma_v^{U^1})(x)f^{U^2}(x)dx+\int_{\underline{v}}^{\overline{v}}\gamma_v^{U^1}(x)(f^{U^2}-f^{U^1})(x)dx}{\left(\eta+\int_{\underline{v}}^{\overline{v}}\gamma_v^{U^1}(x)f^{U^1}(x)dx\right)\left(\eta+\int_{\underline{v}}^{\overline{v}}\gamma_v^{U^2}(x)f^{U^2}(x)dx\right)}.$$

When U^2 and U^1 are close, $max\{U_v^2,x\}$ and $max\{U_v^1,x\}$ are close, and, by Lemma 6, f^{U^2} is close to f^{U^1}. Moreover, by the proof of Lemma 8 in Smith (2006), part C, if U^2 and U^1 are sufficiently close, then $\sigma_v^{U^1}$ and $\sigma_v^{U^2}$ are close.[19] Therefore, by definition, $\gamma_v^{U^1}$ and $\gamma_v^{U^2}$ are close as well. Since η, U^i, and f^{U^i} are are uniformly bounded by some constant $C>0$, we can conclude that $|\int_I T(U_v^1)-T(U_v^2)dv|$ is small whenever $|\int_I U_v^1-U_v^2dv|$ is small for any $I\subseteq[\underline{v},\overline{v}]$. ■

Thus, T satisfies the requirements for Schauder's fixed-point theorem (see Istrăescu, 1981, Theorem 5.1.3). That is, there exists a $U^\star$ such that $T(U^\star)=U^\star$.

Proof of Proposition 8. In the first part of the proof, we use the balanced flow condition together with the requirement that agents' strategies be optimal to establish that $U_{\overline{v}}^\star$ goes to $\overline{v}$ when μ goes to infinity.

LEMMA 9: $lim_{\mu\to\infty}U_{\overline{v}}^\star=\overline{v}$.

[19] This is a step in showing that the map from value functions to match indicator functions is continuous.

Proof. Recall that, in equilibrium, $[\hat{a}_{\overline{v}}, \overline{v}]$ is the top class. The optimality of $\overline{v}$'s strategy implies that

$$\hat{a}_{\overline{v}} \geq \frac{\int_{\hat{a}_{\overline{v}}}^{\overline{v}} f(x)x dx}{\eta + \int_{\hat{a}_{\overline{v}}}^{\overline{v}} f(x)dx}. \tag{A10}$$

Rearranging yields

$$\hat{a}_{\overline{v}} \frac{r+\delta}{\mu(F(\overline{v}) - F(\hat{a}_{\overline{v}}))} \geq \frac{\int_{\hat{a}_{\overline{v}}}^{\overline{v}} f(x)x dx}{F(\overline{v}) - F(\hat{a}_{\overline{v}})} - \hat{a}_{\overline{v}}. \tag{A11}$$

The balanced-flow condition implies that within a class, the steady-state distribution F is a rescaling of the original distribution G. Thus,

$$\frac{\int_{\hat{a}_{\overline{v}}}^{\overline{v}} f(x)x dx}{F(\overline{v}) - F(\hat{a}_{\overline{v}})} = \frac{\int_{\hat{a}_{\overline{v}}}^{\overline{v}} g(x)x dx}{G(\overline{v}) - G(\hat{a}_{\overline{v}})} = E_G[w|w > \hat{a}_{\overline{v}}]. \tag{A12}$$

Moreover, the balanced-flow condition implies that

$$(F(\overline{v}) - F(\hat{a}_{\overline{v}}))(\delta + \mu(F(\overline{v}) - F(\hat{a}_{\overline{v}}))) = \beta(1 - G(\hat{a}_{\overline{v}})). \tag{A13}$$

It is now possible to see that $G(\hat{a}_{\overline{v}})$ goes to 1 when μ goes to infinity. Otherwise, by (A13), $\mu(F(\overline{v}) - F(\hat{a}_{\overline{v}}))$ goes to infinity when μ goes to infinity, which implies that the LHS of (A11) converges to 0. This, in turn, implies that $E_G[w|w > \hat{a}_{\overline{v}}] - \hat{a}_{\overline{v}}$ goes to 0, which violates the assumption that $G(\hat{a}_{\overline{v}})$ does not go to 1 when μ goes to infinity. Hence, $\hat{a}_{\overline{v}}$ goes to $\overline{v}$, and so $U^\star_{\overline{v}}$ goes to $\overline{v}$. ■

Let $\epsilon > 0$ and $v, w \in (\underline{v}, \overline{v})$ such that $v + \epsilon < w$. If v marries in equilibrium, then

(A14)

$$v \geq U^\star_v \geq \frac{\psi F(\overline{v}) p(a_v)(1 - p(w)) \frac{\int_w^{\overline{v}} f(x)x dx}{F(\overline{v}) - F(w)}}{\eta + \psi F(\overline{v}) p(a_v)(1 - p(w))} > \frac{\psi F(\overline{v}) p(a_v)(1 - p(w)) w}{\eta + \psi F(\overline{v}) p(a_v)(1 - p(w))},$$

where $p(z) = F(z)/F(\overline{v})$. Note that the second inequality above follows from the fact that the acceptance cutoff $\hat{a}_v = w$ is not necessarily optimal. Observe that if

$$p(a_v)(1 - p(w)) F(\overline{v}) \mu \tag{A15}$$

goes to infinity when μ goes to infinity, then (A14) is violated for a sufficiently large μ.

Assume that there are two agents, v_1 and v_2, who marry in equilibrium such that $\underline{v} + \epsilon < v_1 < v_2 < \overline{v} - \epsilon$ and $v_2 > v_1 + \epsilon$.

As we showed in the above lemma, $\hat{a}_{\overline{v}}$ goes to $\overline{v}$ when μ goes to infinity. Thus, for a sufficiently large μ, (A11) must hold in equality and, therefore, $\mu(1-p(\hat{a}_{\overline{v}}))F(\overline{v})$ goes to infinity when μ goes to infinity. Thus, if we set $w = \hat{a}_{\overline{v}}$ and $v = v_2$, we obtain from (A15) that $p(a_{v_2})$ goes to 0 when μ goes to infinity. Since v_2 marries, $v_2 \leq a_{v_2}$ and, therefore, $p(v_2)$ goes to 0 as well.

We now repeat this exercise with $v = v_1$ and $w = v_2$ and show that $p(v_2)$ goes to 1 when μ goes to infinity. By monotonicity, $a_z \leq a_{v_1}$ for all $z < v_1$. Therefore, every $z < v_1$ leaves the market at a rate no greater than $\delta + \mu F(a_{v_1})$ (recall that $v_1 \leq a_{v_1}$ since v_1 marries) and so, the balanced-flow condition implies that

$$F(v_1)[\delta + \mu F(a_{v_1})] \geq \beta G(v_1) >> 0.$$

By the above balanced-flow condition, $\mu F(a_{v_1})$ must go to infinity when μ goes to infinity, as otherwise the LHS goes to 0. From (A15), $1 - p(v_2)$ must go to zero, in contradiction to $p(v_2)$ going to zero. We can conclude that for a sufficiently large μ, at most 3ϵ of the pizzazz values marry.

Market Segmentation

We now show when agents are partially cursed, segmentation can increase the share of agents who marry in equilibrium.[20] Formally, let $v_n = \underline{v} < v_{n-1} < ... < v_1 < v_0 = \overline{v}$ represent a partition of the market into n cells and assume that men in the i-th cell meet women in the i-th cell at a flow rate $\mu(F(v_{i-1}) - F(v_i))$ and vice versa. Agents of different cells do not meet each other.

PROPOSITION 9: *In the segmented market, the share of agents who marry in finite time is higher than in the non-segmented market.*

Proof. To prove this result, we compare $U(v, v)$ in the original market with $U(v, v)$ in the segmented market and show that the latter is smaller. Recall that $U(v, v)$ in the original market is given in (5). First, assume that $n = 2$ and partition the market into two segments, an upper segment $[v^1, \overline{v}]$ and a lower segment $[\underline{v}, v^1)$. Denote the mass of agents in each segment by $1 - m$ and m, respectively. Thus,

$$U^{seg}(v, v) = \begin{cases} \frac{\psi \frac{F(v)-m}{1-m}(1-F(v))E[w|w>v]}{\eta+\psi \frac{F(v)-m}{1-m}(1-F(v))} & \textit{for } v \in [v^1, \overline{v}] \\ \frac{\psi \frac{F(v)}{m}(m-F(v))E[w|v^1>w>v]}{\eta+\psi \frac{F(v)}{m}(m-F(v))} & \textit{for } v \in [\underline{v}, v^1). \end{cases}$$

Hence, $U^{seg}(v, v) \leq U(v, v)$ for every $v \in [\underline{v}, \overline{v}]$. That is, our condition for marriage in finite time holds for more agents in the segmented market than in the

[20] It is possible to obtain the same result in the analogy-based expectation equilibrium framework.

unsegmented one. We can now partition each of the two segments into subsegments and obtain a similar result: the share of agents who marry in finite time in each subsegment is greater than in the unsegmented market. ■